EXERCISES 10: LECTURE FOUNDATIONS OF MATHEMATICS

Exercise 1. As a reminder: a total order on a set X is called well-ordered from below (above) if there exists a smallest (biggest) element for any non-empty subset of X with respect to the fixed total order.

Let X a totally ordered set which is well-ordered from below and above. Show that X is finite.

Exercise 2. Find an order on $\mathbb{Q}$ such that $\mathbb{Q}$ is well-ordered from below.

Exercise 3. Let X a set. Define

$$\Delta\colon \mathfrak{P}(X) \times \mathfrak{P}(X) \to \mathfrak{P}(X),\ \Delta(A,B) = (A \cup B) \setminus (A \cap B),$$
$$\cap\colon \mathfrak{P}(X) \times \mathfrak{P}(X) \to \mathfrak{P}(X),\ \cap(A,B) = A \cap B.$$

Show that $(\mathfrak{P}(X), \Delta, \cap)$ is a unital ring.

Exercise 4. Let $\mathbb{Z}^{\mathbb{N}}$ be the set of all maps $\mathbb{N} \to \mathbb{Z}$. For $f, g \in \mathbb{Z}^{\mathbb{N}}$ define an addition $+$ and a multiplication $*$ via

$$+\colon \mathbb{Z}^{\mathbb{N}} \times \mathbb{Z}^{\mathbb{N}} \to \mathbb{Z}^{\mathbb{N}},\ +(f,g)(x) = f(x) + g(x),$$
$$*\colon \mathbb{Z}^{\mathbb{N}} \times \mathbb{Z}^{\mathbb{N}} \to \mathbb{Z}^{\mathbb{N}},\ *(f,g)(x) = \sum_{ab=x} f(a)g(b),$$

where the sum runs over all $a, b \in \mathbb{N}$ with $ab = x$. Show that $(\mathbb{Z}^{\mathbb{N}}, +, *)$ is a commutative, unital ring.

Submission of the exercise sheet: 03.Dec.2018 before the lecture. **Return of the exercise sheet**: 06.Dec.2018 during the exercise sessions.