EXERCISES 8: LECTURE FOUNDATIONS OF MATHEMATICS

Exercise 1. Let X and Y be finite sets. Decide (with a proof) how many injective maps $X \to Y$ exist.

Exercise 2. Show that a set $X \neq \emptyset$ is countable if and only if there is a surjection from $\mathbb{N}_0$ to X.

Exercise 3. Let X be a countable set. Show that the set of all finite subsets of X is countable.

Exercise 4. Let X be a set. Show that the following statements are equivalent.

(i) X is infinite.

(ii) For all maps $f\colon X \to X$ there exists $\emptyset \subsetneq A \subsetneq X$ with $f(A) \subset A$.

Hint: Take $f\colon \{0, 1, \ldots, n\} \to \{0, 1, \ldots, n\}, f(i) = i + 1$ where $n + 1$ should be considered as 0. Does it satisfy (ii)? Moreover, show that (ii) holds for $X = \mathbb{N}_0$ and reduce the general case to this situation.

Submission of the exercise sheet: 19.Nov.2018 before the lecture. **Return of the exercise sheet**: 29.Nov.2018 during the exercise sessions.