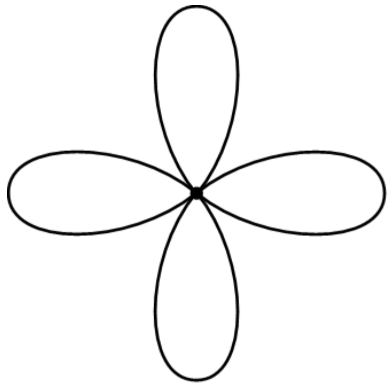
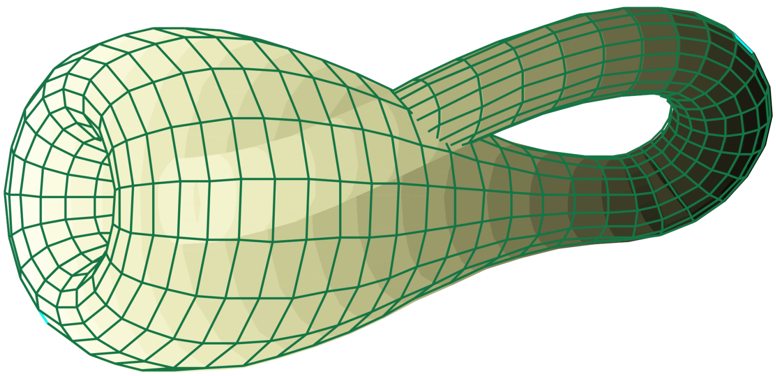
ASSIGNMENT 1: LECTURE ALGEBRAIC TOPOLOGY

Exercise 1. An n rose, or a bouquet of n circles, is $\bigvee_{i=1}^{n} S^1$, *e.g.* for $n = 4$:



Show that any connected, finite graph is homotopy equivalent to an n rose for some n.

Exercise 2. Let X be the subset of $\mathbb{R}^3$ given by the most common immersion of the Klein bottle into $\mathbb{R}^3$ (we consider X as a subset of $\mathbb{R}^3$ and not as the Klein bottle itself):

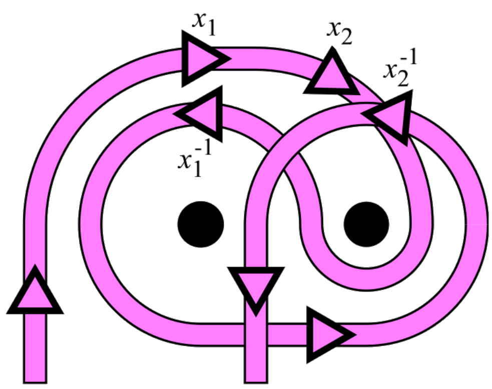


Show, *e.g.* by drawing the relevant pictures, that $X \simeq S^1 \vee S^1 \vee S^2$.

Exercise 3. Compute $\pi_1(S^1 \vee S^1 \vee S^1)$ and solve the following variant of Spivak's hanging-pictures-puzzle: "Hang a picture on three nails so that removing any two nails fells the picture, but removing any one nail leaves the picture hanging.".

Hint: The solution to the original puzzle "Hang a picture on two nails so that removing any nail fells the picture." in algebraic notation is

$\rightsquigarrow$

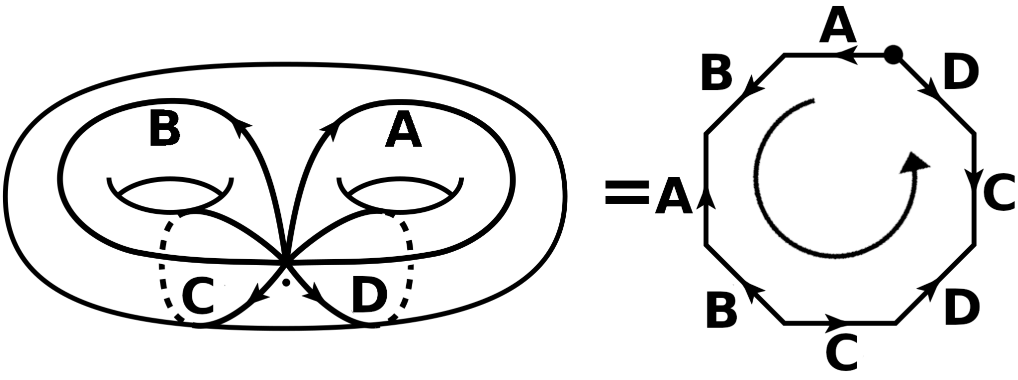


$\rightsquigarrow$ $x_2^{-1}x_1^{-1}x_2x_1$.

Exercise 4. Let $M_{g,0}$ be the surface of genus g and with no boundary. Compute $\pi_1(M_{g,0})$ for $g > 0$.

Addendum:

- You can assume that $M_{g,0}$ is defined via its fundamental polygon obtained by identifying edges of a $4g$-gon as in the picture below.
- Hint:



- The first assignment is due 17.Sep.2021, latest 11:59pm.
- Please upload your answers to Canvas.
- The material from the first four lectures can be used freely, including the relevant sections in Hatcher.