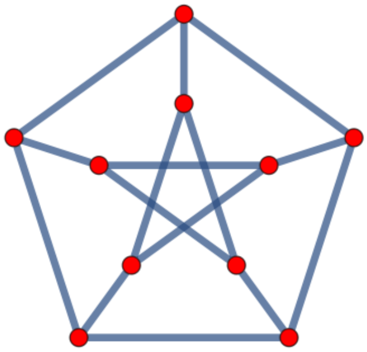
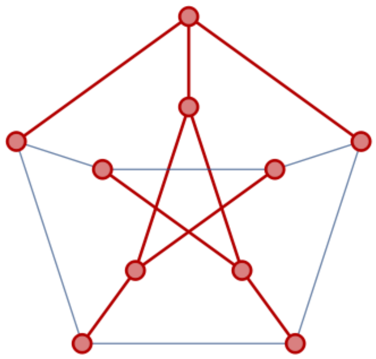
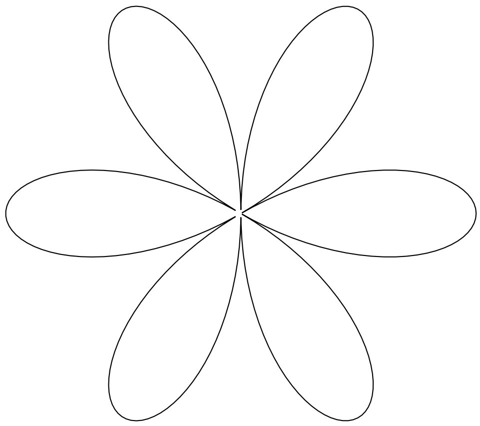
ASSIGNMENT 2: LECTURE ALGEBRAIC TOPOLOGY

Exercise 1. Compute the homology $H_*(G)$ of the Petersen graph G:

$G =$ 

Can you guess what the homology of a general graph is?
Hint: The following two pictures should be helpful.





Exercise 2. Classify the Platonic solids by using that they are cell complexes for the sphere S^2 and that $\chi(S^2) = 2$.

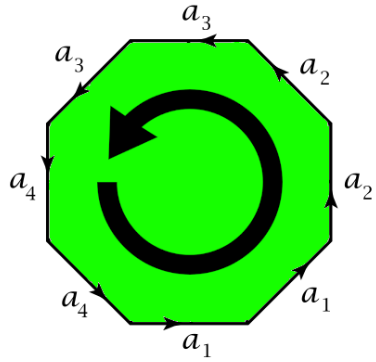
Addendum:

- ▶ Note that Platonic solids have a definition and are not arbitrary polyhedra: they are convex regular polyhedron in $\mathbb{R}^3$.
- ▶ Hint: We know the answer, so let us make a table where m, n are defined by $mV = 2E = nF$:

	m	n	V	E	F
Tetrahedron	3	3	4	6	4
Cube	3	4	8	12	6
Octahedron	4	3	6	12	8
Dodecahedron	3	5	20	30	12
Icosahedron	5	3	12	30	20

Observe that $\frac{1}{2} < \frac{1}{m} + \frac{1}{n}$ holds.

Exercise 3. For $g \geq 1$ let $M^-_{g,0}$ denote the closed non-orientable surface of genus g defines via its fundamental polygon, *i.e.* a $2g$-sided polygon with attaching word $a_1^2 \dots a_g^2$. For example, for $g = 4$ we have:

$M^-_{4,0} \rightsquigarrow$ 

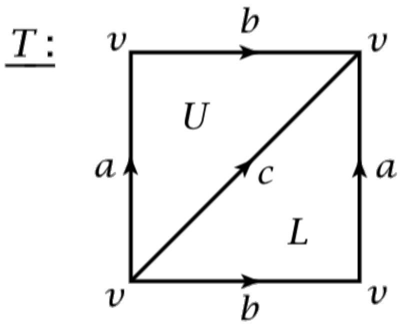
Compute the homology $H_*(M^-_{g,0})$ and the Hilbert–Poincare polynomial $P(M^-_{g,0})$.

Hint: Note that $M^-_{1,0} \cong \mathbb{R}P^2$ and $M^-_{2,0}$ is the Klein bottle, and recall how to calculate their homologies. (Beware that the above are not the standard presentations of these two surfaces: a surface can be defined by different fundamental polygons.)

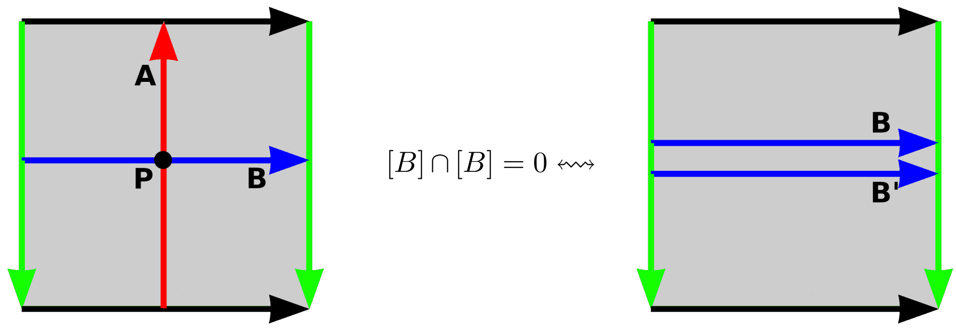
Exercise 4. Compute the cohomology ring $H^\bullet(T)$ of the torus T from the definitions (*i.e.* not going to the intersection ring).

Addendum:

- You can assume that T is defined via the following simplicial structure:



- Hint: The main calculations in the intersection ring are

$[A] \cap [B] = [P] \rightsquigarrow$ $[B] \cap [B] = 0 \rightsquigarrow$ 

The main point is to find expressions of $[A]$ and $[B]$ in $C^*(T)$. It is then not hard to verify that the intersection calculation is reflected in singular cohomology.

- The second assignment is due 05.Nov.2021, latest 11:59pm.
- Please upload your answers to Canvas.
- The material from all lectures can be used freely, including the relevant sections in Hatcher.