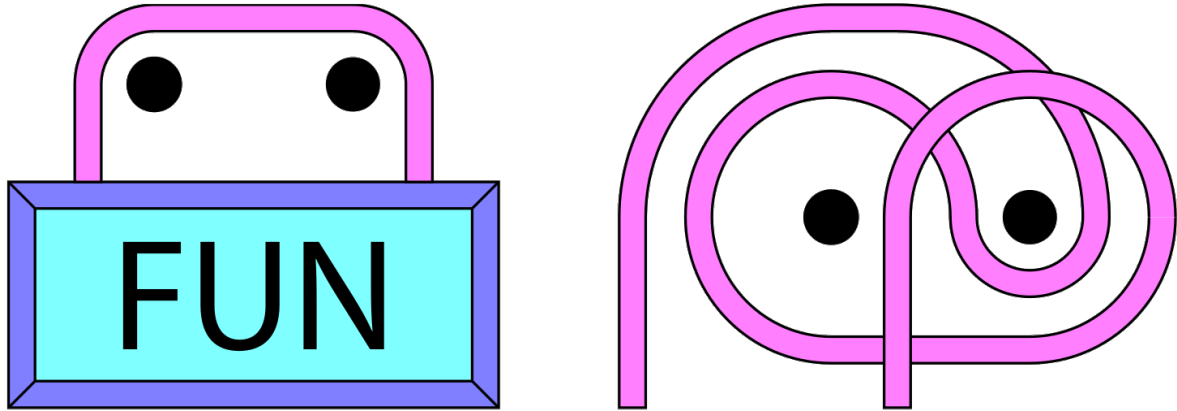


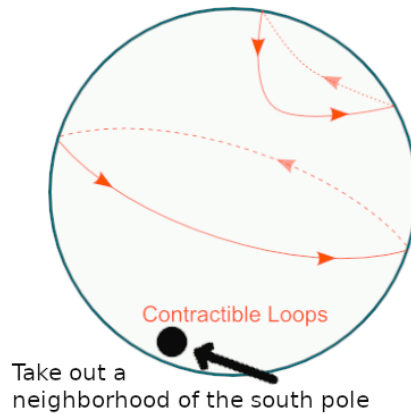
## EXERCISES 4: LECTURE ALGEBRAIC TOPOLOGY

**Exercise 1.** Let  $D_2^2$  be the disc with two holes. Interpret the two pictures

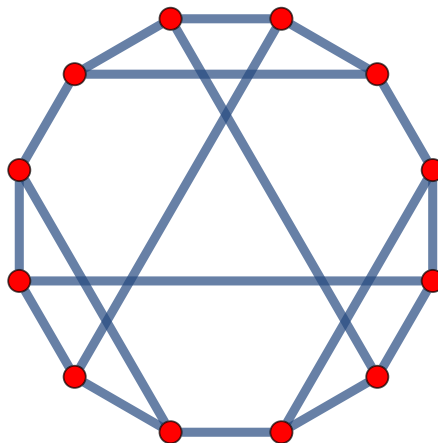


as elements of  $\pi_1(D_2^2)$ . What happens if we close one hole?

**Exercise 2.** Compute  $\pi_1(S^2)$ . Hint:



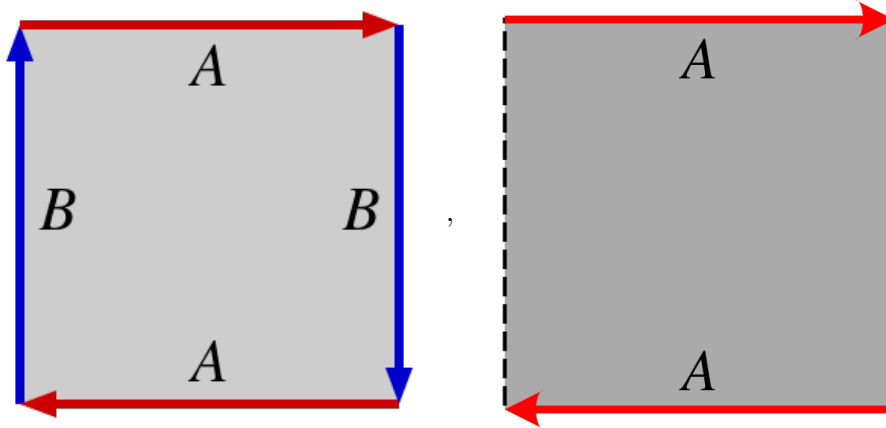
**Exercise 3.** Compute  $\pi_1(X)$  for the following cell complex:



**Exercise 4.** Compute  $\pi_1(\mathbb{R}P^2)$  for the real projective plane  $\mathbb{R}P^2$ .

Addendum:

- Here are the fundamental polygons of  $\mathbb{R}P^2$  (left) and the Möbius strip (right):



- Hint: [www.math3ma.com/blog/the-fundamental-group-of-the-real-projective-plane](http://www.math3ma.com/blog/the-fundamental-group-of-the-real-projective-plane)

- The exercises are optimal and not mandatory. Still, they are highly recommend.
- There will be 12 exercise sheets, all of which have four exercises.
- The sheets can be found on the homepage [www.dtubbenhauer.com/lecture-algtop-2021.html](http://www.dtubbenhauer.com/lecture-algtop-2021.html).
- If not specified otherwise, spaces are topological space, maps are continuous *etc.*
- There might be typos on the exercise sheets, my bad, so be prepared.