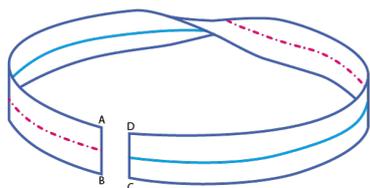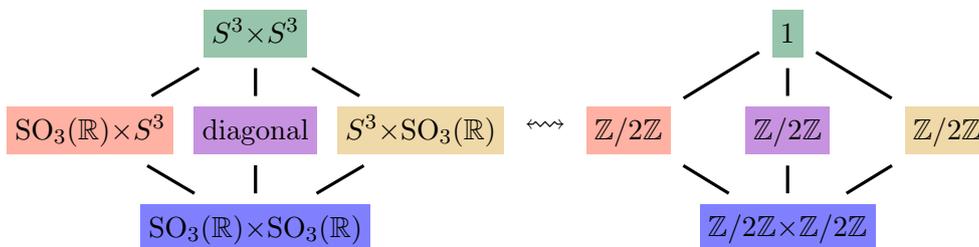# EXERCISES 5: LECTURE ALGEBRAIC TOPOLOGY

Exercise 1. Find all connected covering spaces of the circle S^1 by using that $\pi_1(S^1) \cong \mathbb{Z}$.

Exercise 2. Find a 2-fold cover of the Möbius strip (and build both at home). Hint:

Annulus: Glue $A - C$ and $B - D$,
Möbius strip: Glue $A - D$ and $B - C$.

Exercise 3. Explain (meaning understand the ingredients) the following illustration:
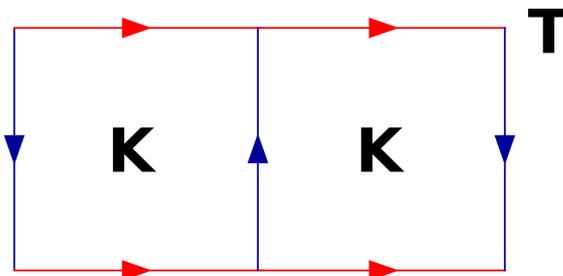

Addendum:
- ▶ S^3 is the three-dimensional sphere, and $SO_3(\mathbb{R})$ is the special orthogonal group acting on $\mathbb{R}^3$ (the rotation group).
- ▶ Hint: math.stackexchange.com/questions/123650

Exercise 4. Show explicitly that the torus T is a 2-fold cover of the Klein bottle K.
Addendum:
- ▶ Observe that

$$\mathbb{Z}^2 \cong \pi_1\left(T = \text{[torus]}\right) \subset_{\text{subgroup}} \pi_1\left(K = \text{[Klein bottle]}\right) \cong \langle a, b \mid abab^{-1}\rangle$$

- ▶ Hint: math.stackexchange.com/questions/1073425

- ▶ The exercises are optimal and not mandatory. Still, they are highly recommend.

▶ There will be 12 exercise sheets, all of which have four exercises.

▶ The sheets can be found on the homepage www.dtubbenhauer.com/lecture-algtop-2021.html.

▶ If not specified otherwise, spaces are topological space, maps are continuous *etc.*

▶ There might be typos on the exercise sheets, my bad, so be prepared.