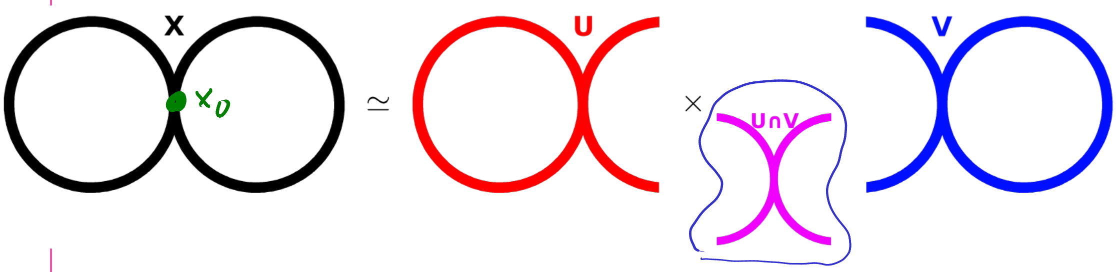
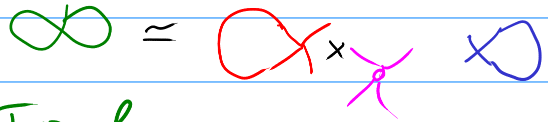
Seifert - van Kampen!





Topology

$$X \simeq U \times_{U \cap V} V$$

Algebra

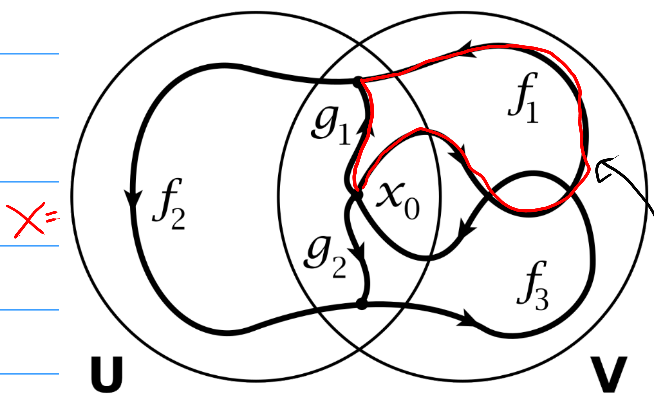
free product wrt K

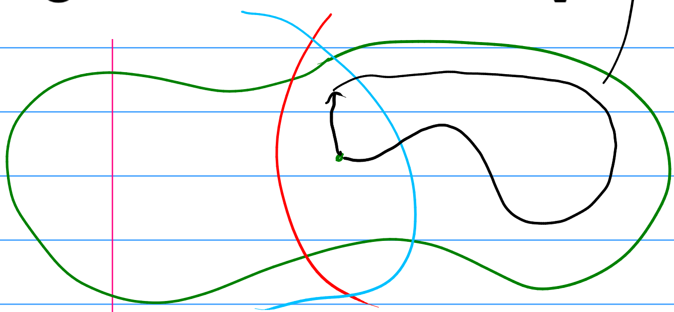
$$\pi_1(X) \simeq \pi_1(U) *_{\pi_1(U\cap V)} \pi_1(V)$$

K

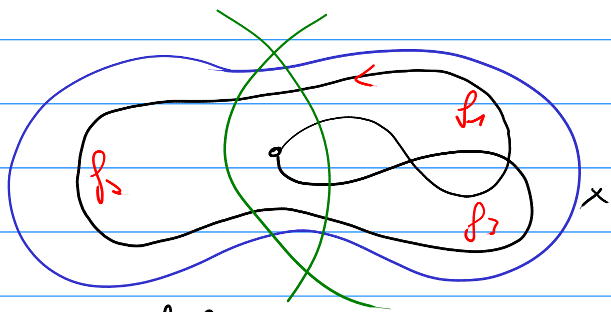
??

Lemma 1.15. *If a space X is the union of a collection of path-connected open sets A_α each containing the basepoint $x_0 \in X$ and if each intersection $A_\alpha \cap A_\beta$ is path-connected, then every loop in X at x_0 is homotopic to a product of loops each of which is contained in a single A_α.*





$X = U \cup V$



$f_3 f_2 f_1$

$(f_3 g_1)(g_1^{-1} f_2 g_1)(g_1^{-1} f_1)$

U V

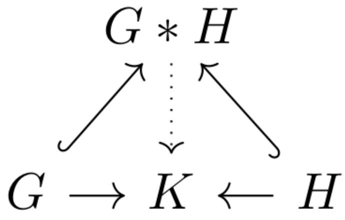
Question: How is Lemma 1.15 reflected in Algebra??

Given two groups G and H, construct a group $G * H$ by demanding that:

(a) G, H are subgroups of $G * H$

(b) $G * H$ is generated by G, H

(c) Any two homomorphisms from G and H into a group K factor uniquely through a homomorphism from $G * H$ to K



$\pi_1(U) * \pi_1(V) \twoheadrightarrow \pi_1(X)$

$g * g' \longleftarrow f$

$G * H$ "free product of G and H"

$G * H$ exists and is uniquely determined by these properties

If $G = \langle S_G \mid R_G \rangle, H = \langle S_H \mid R_H \rangle$, then $G * H = \langle S_G \cup S_H \mid R_G \cup R_H \rangle$

$G * H$ has the relations of G, H and nothing more

$s_i\, s_j\, s_k \in S$

$G = \langle S_G \mid R_G \rangle$

set $S = \{s_1, \dots, s_n\}$ "alphabet"

$\underbrace{s_i\, s_j\, s_k \cdots}_{\text{finitely many}}$

group structure $w \circ w' = ww'$

e.g. $(s_1 s_2) \circ (s_2 s_1) = s_1 s_2 s_2 s_1$

The file crushed here and my save got lost, sorry for that! Please check the video for the slides.