RESEARCH STATEMENT

DANIEL TUBBENHAUER

Abstract. This is my research statement for the year 2017 containing a brief, general overview, and some introductory words for some of my most recent research questions and projects. (Finished or still work in progress.)

Please note. My intention is not to give rigorous mathematical definitions or statements, but rather to give an informal overview about my research. I hope the reader will forgive me some of my sloppy formulations.

Contents

In short

My main research interest is 2-categorical representation theory (of e.g. "categorified" Coxeter groups), categorification (of e.g. quantum groups) and applications in representation theory, low-dimensional topology and algebraic geometry. In particular, I am interested in algebraic, combinatorial and diagrammatic aspects of categorification. I am also interested in related topics as for example representation theoretical questions about Hecke/Brauer algebras or Lie groups and modular representation theory.

My research interest at the moment basically splits into a "topologically motivated" part concerning algebraic constructions of link homologies and their functoriality, see Section 2.2, as well as a "representation theory motivated" part concerning "higher" representation of Coxeter groups, see Section 2.3, or questions coming from invariant theory, see Section 2.1.

1. General overview

Before going into the details of my current research, let me try to motivate the basic questions which play a major role in all of my research interests.

1.1. Categorification.

1.1.1. *Seeking "higher" structure.* The notion *categorification* was introduced by Crane [Cra95] based on an earlier joint work with Frenkel [CF94].

But the concept of categorification has a much longer history than the word itself. Forced to explain the concept in one sentence, I would choose

"Interesting integers are shadows of richer structures in categories."

The basic idea is as follows. Take a "set-based" structure S and try to find a "category-based" structure $\mathbf{C}$ such that S is just a shadow of the category $\mathbf{C}$. If the category $\mathbf{C}$ is chosen in a "good" way, then one has an explanation of facts about the structure S in a categorical language. That is, certain facts in S can be explained as special instances of natural constructions.

Experience tells us that the categorical structure does not only explain properties of the "set-based" structure, but is usually much richer and more interesting itself.

Remark

In principal, one can perform such a "categorification process" on any level, e.g. one can categorify an "n-category like structure" into an "n+1-category like structure". Without going into any details about higher categories, the slogan for me is that a set is a collection of "number like structures" with the set of natural numbers as a prototypical example; a category is a collection of "set like structures" with the category of sets as a prototypical example; a 2-category is a collection of "category like structures" with the 2-category of categories as a prototypical example; a 3-category is a collection of "2-category like structures"...

1.1.2. *Decategorification.* Categorification comes with an "inverse" called *decategorification*; and categorification can be seen as "remembering" or "inventing" information while decategorification is more like "forgetting" or "identifying" structure, which is – of course – easier.

Remark

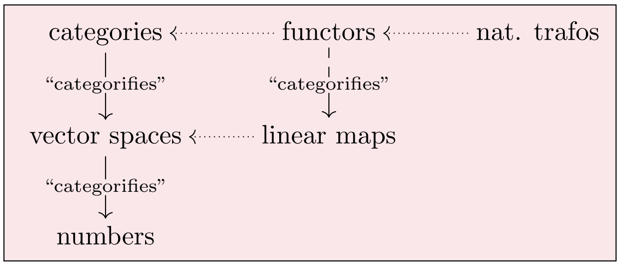
One has to specify what is meant by decategorification before talking saying "We have categorified XY". However, I will be a bit sloppy in what follows.

A blueprint example is that the category of $\mathbb{K}$-vector spaces $\mathbb{K}\mathbf{Vect}$ (over some field $\mathbb{K}$) "categorifies" $\mathbb{Z}_{\geq 0}$. In this case the decategorification is given by taking dimensions.

Another "standard" example is to take (a suitable version of) the Grothendieck group $[\cdot]_{\mathbb{K}} = K_0(\cdot) \otimes_{\mathbb{Z}} \mathbb{K}$. (For example $K_0(\mathbb{K}\mathbf{Vect}) \cong \mathbb{Z}$ and $[\mathbb{K}\mathbf{Vect}]_{\mathbb{K}} \cong \mathbb{K}$.)

In fact, if we think of the category $\mathbb{K}\mathbf{Vect}$ as being a "set-based" structure (with objects being sets with a $\mathbb{K}$-linear structure), then we might want to categorify this further by considering the 2-category $\mathbb{K}\mathfrak{Cat}$ of $\mathbb{K}$-linear categories, $\mathbb{K}$-linear functors and $\mathbb{K}$-linear natural transformations. Taking an appropriate 2-categorical Grothendieck group recovers $\mathbb{K}\mathbf{Vect}$.

1.1.3. *One diagram is worth a thousand words.* Each step of a "categorification process" should reveal more structure. An illustration for the example from above is the following (omitting the $\mathbb{K}$):



Here we first "categorify" numbers into $\mathbb{K}$-vector spaces. The new information available are now $\mathbb{K}$-linear maps between $\mathbb{K}$-vector spaces. (Thus, we have the whole power of linear algebra at hand.) There is no reason to stop: we can "categorify" $\mathbb{K}$-vector spaces into $\mathbb{K}$-categories, $\mathbb{K}$-linear maps into $\mathbb{K}$-linear functors. Again, we see a new layer of information, namely the natural transformations between these functors.

1.1.4. *Examples of categorification.* The following list of example is already long, but biased and far from being complete. Much more can be found in the work of Baez–Dolan [BD98], [BD01] for examples which are related to more combinatorial parts of categorification, or Crane–Yetter [CY98], Khovanov–Mazorchuk–Stroppel [KMS09], Mazorchuk [Maz12] or Savage [Sav14] for examples from algebraic categorification.

▷ In some sense the "most classical, but quite recent" example is Khovanov's categorification of the Jones (or $\mathfrak{sl}_2$) polynomial [Kho00].

▷ Khovanov's construction can be extended to a categorification of the Reshetikhin-Turaev $\mathfrak{sl}_n$-link polynomial and the HOMFLY–PT polynomial, e.g. see [KR08]. Moreover, some "applications" of Khovanov's categorification are:

  ▶ It is functorial, e.g. see [Cap08], [CMW09], [Bla10] or [ETW17]: it "knows" about link cobordisms. Since such cobordisms are cobordisms embedded in the four-space, this gives a way to get information about smooth structures in dimension 4. (And 4-dimensional, smooth topology is hell!)

  ▶ Kronheimer–Mrowka showed [KM11], by comparing Khovanov homology to Knot Floer homology, that Khovanov homology detects the unknot. This is still an open question for the Jones polynomial.

  ▶ Rasmussen obtained his famous invariant by comparing Khovanov homology to a variation of it. He used his invariant to give a combinatorial proof of the Milnor conjecture [Ras10]. Note that he also gives a way to construct exotic $\mathbb{R}^4$ from his approach [Ras05].

  ▶ There is a variant of Khovanov homology, called odd Khovanov homology – see [ORS13] – which differs over $\mathbb{Q}$ and can not be seen on the level of polynomials.

  ▶ There is a variant that categorifies the HOMFLY–PT polynomial. This categorification is a "rich" structure itself and has a lot of connections to various parts of mathematics and related fields, see e.g. [GORS14] and the references therein.

- ▶ Not the main point but: it is strictly stronger than the Jones polynomial.

▷ Other notable categorifications related to low-dimensional topology are:

- ▶ Floer homology can be seen as a categorification of the Casson invariant of a manifold. Floer homology is "better" than the Casson invariant, e.g. it is possible to construct a 3+1 dimensional TQFT which for closed four-dimensional manifolds gives Donaldson's invariants, see for example [Wit12].

- ▶ Knot Floer homology can be seen as a categorification of the classical knot invariant of Alexander–Conway, see for example [OS04].

- ▶ The approach to categorify the Reshetikhin–Turaev $\mathfrak{g}$-polynomial for arbitrary simple Lie algebra $\mathfrak{g}$ by Webster [Web13].

▷ The notion categorification is from the interplay of low-dimensional topology and representation theory. Hence, there are also several examples coming from representation theory as e.g.:

- ▶ Ariki gave [Ari96] a remarkable categorification of all finite-dimensional, irreducible representation of $\mathfrak{sl}_n$ for all n as well as a categorification of integrable, irreducible representations of the affine version $\widehat{\mathfrak{sl}}_n$. In short, he identified the Grothendieck group of blocks of so-called Ariki–Koike cyclotomic Hecke algebras with weight spaces of such representations in such a way that direct summands of induction and restriction functors between cyclotomic Hecke algebras for $m, m+1$ act on the K_0 as the E_i, F_i of $\mathfrak{sl}_n$.

- ▶ Chuang–Rouquier [CR08] masterfully used the categorification of good old $\mathfrak{sl}_2$ to solve an open problem in modular representation theory of the symmetric group using a completely new approach.

- ▶ Khovanov–Lauda [KL10], and – independently – Rouquier [Rou08] have categorified all quantum Kac–Moody algebras with their canonical bases.

- ▶ Khovanov–Qi [KQ15] and Elias–Qi [EQ16] have an approach how to categorify at roots of unity. Their categorification of quantum $\mathfrak{sl}_2$ for the quantum parameter q being a (certain type of) root of unity can be (the future will prove me right or wrong) the first step to categorify the Witten–Reshetikhin–Turaev invariants of 3-manifolds.

- ▶ The so-called Soergel category $\mathfrak{S}$ can be seen in the same vein as a categorification of the Hecke algebras in the sense that the split Grothendieck group gives the Hecke algebras. We note that Soergel's construction shows that Kazhdan–Lusztig bases have positive integrality properties, see [Soe90] and related publications. Indeed, this approach was masterfully carried out by Elias–Williamson who finally proved the Kazhdan–Lusztig basis conjecture for all Coxeter types [EW14].

▷ Categorifications are also studied in physics, e.g.:

- ▶ In conformal field theory (CFT) researchers study fusion algebras, e.g. the Verlinde algebra. Examples of categorifications of such algebras are known, e.g. using categories connected to the representation theory of quantum groups at roots of unity [Kho16], and contain more information than these algebras, e.g. the R-matrix and the quantum $6j$-symbols.

- ▶ The Witten genus of certain moduli spaces can be seen as an element of $\mathbb{Z}[[q]]$. It can be realized using elliptic cohomology, see e.g. [AHS01].

1.2. **Higher representation theory.**

1.2.1. *The "classical" question.*

"Groups, as men, will be known by their actions." – Guillermo Moreno

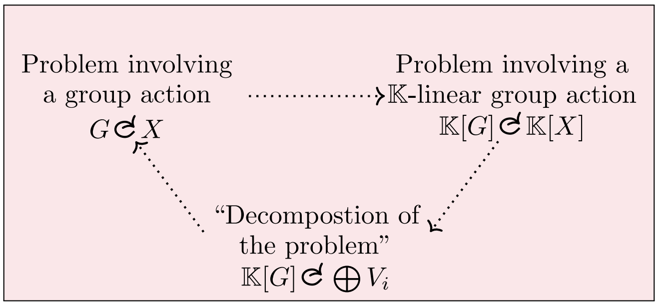
The study of group actions is of fundamental importance in mathematics and related field. Sadly, it is also very hard. Luckily, Frobenius ($\sim$1895 onwards), Burnside ($\sim$1900 onwards) and many others gave us the following.

Let $\mathbb{K}[G]$ be the group ring of a (finite) group G. Representation theory is the study of $\mathbb{K}$-linear group actions:

$$\mathrm{R}\colon \mathbb{K}[G] \longrightarrow \mathrm{End}(V), \quad \mathrm{R}(g) = \text{a "matrix" in } \mathrm{End}(V),$$

with V being some $\mathbb{K}$-vector space. We call V a G-module or a G-representation.

Representation theory approach: the analogous $\mathbb{K}$-linear problem of classifying G-modules has a satisfactory answer for many groups.



Thus, given a group G (or a ring, an algebra etc.), a classical and very interesting question is:

"Can we describe the symmetries G can act on, i.e. its representation theory?"

1.2.2. *"Categorified" representation theory.*

The related, categorical question that arises is whether we can categorify the classical notions. That is:

"Can we describe the symmetries a category $\mathbf{C}$ can act on,
i.e. its representation theory?"

Moreover, the next question would be if we can categorify this again:

"Can we describe the symmetries a 2-category $\mathfrak{C}$ can act on,
i.e. its 2-representation theory?"

I will give a short introduction to the basic ideas. Much more details can, for example, be found in Rouquier's paper [Rou08]. Another also very nice introduction is the book of Mazorchuk [Maz12].

Let A be some (group) algebra, V be an A-module and $\mathbf{V}$ be a (suitable) category. Let $2\mathbf{End}(\mathbf{V})$ denotes its associated 2-category of endofunctors. Then:

$$\begin{array}{ccc}
\text{“Classical”} & \rightsquigarrow & \text{“Higher”} \\
a \mapsto \mathrm{R}(a) \in \mathrm{End}(V) & \rightsquigarrow & a \mapsto \boldsymbol{\mathcal{R}}(a) \in 2\mathbf{End}(\mathbf{V}) \\
(\mathrm{R}(a_1)\cdot \mathrm{R}(a_2))(v) = \mathrm{R}(a_1a_2)(v) & \rightsquigarrow & (\boldsymbol{\mathcal{R}}(a_1)\circ \boldsymbol{\mathcal{R}}(a_2))\binom{X}{\alpha} \cong \boldsymbol{\mathcal{R}}(a_1a_2)\binom{X}{\alpha}
\end{array}$$

A *(weak) categorification of the* A*-module* V should be thought of as a categorical action of A on $\mathbf{V}$ with an isomorphism ψ such that

$$\begin{array}{ccc}
[\mathbf{V}]_{\mathbb{K}} & \xrightarrow{[\boldsymbol{\mathcal{R}}_a]} & [\mathbf{V}]_{\mathbb{K}} \\
\psi \downarrow & \circlearrowleft & \downarrow \psi \\
V & \xrightarrow[\mathrm{R}(a)]{} & V
\end{array}$$

commutes. Note that such a categorification again “knows” more, i.e. it “knows the relations” between the acting matrices instead of just the acting matrices. But the higher structure is not fixed in such a categorification.

Remark

In a lot of cases there is also a “honest higher layer”. Instead of having the algebra A acting on the $\mathbb{K}$-vector space V, we want it to act on the $\mathbb{K}$-linear category $\mathbf{V}$. But in the usual spirit of categorification this picture should “upgrade”:

$$\begin{array}{ccc}
\mathfrak{A} & \xrightarrow{\text{full-grown 2-action}} & 2\mathbf{End}(\mathbf{V}) \\
[\cdot]_{\mathbb{K}} \downarrow & \overset{\text{categorical action}}{\nearrow} & \downarrow [\cdot]_{\mathbb{K}} \\
\mathrm{A} & \xrightarrow[\text{classical action}]{} & \mathrm{End}(V)
\end{array}$$

Hereby we view the “uncategorified” action as a functor from A to $\mathrm{End}(V)$ (both seen e.g. as categories with one object). Then there should be a categorification $\mathfrak{A}$ of A such that we have a diagram as above – with $\mathfrak{A}$ acting via a 2-functor (which in particular fixes the higher structure).

This – sometimes called *(strong)* 2*-action of* A – works surprisingly often.

1.2.3. *Examples of “categorified representations”.* The following list is again biased. As before, the two sources [KMS09] and [Maz12] give several other examples.

▷ In some sense one of the most classical example of categorified representations is provided by (versions of) the BGG category $\mathcal{O}$.

- ▶ For instance, the regular representation of the group ring $\mathbb{C}[S_n]$ of the symmetric group S_n has a categorification given by projective functors acting on a regular block of the BGG category $\mathcal{O}$ for $\mathfrak{sl}_n$, see e.g. [BG80] (where the authors – of course – do not use the word categorification).

- ▶ Integral Specht modules of S_n can be categorified in a quite similar fashion, see e.g. [KMS08].

- ▶ Similarly for categorifications of the associated Hecke algebra.

- ▶ Other constructions in this spirit are known, see [Maz12] for an overview.

▷ Another nowadays classical categorification is connected to categorification of the tensor product of the vector representation of (quantum) $\mathfrak{sl}_2$:

- ▶ Bernstein–Frenkel–Khovanov [BFK99] categorified the n-fold tensor product of $\mathfrak{sl}_2$ by using certain induction/restriction functors coming from cohomology rings of Grassmannians and flag varieties.

- ▶ Frenkel–Khovanov–Stroppel extended this to the graded setup, and also include a categorification of the highest weight summand of the n-fold tensor product, see [FKS06].

- ▶ Recently Naisse–Vaz [NV16] extended this into a categorification of the Verma modules for $\mathfrak{sl}_2$.

▷ As already mentioned above:

- ▶ Based on joint work with Chuang [CR08], Rouquier [Rou08] (and – independently – Khovanov–Lauda [KL10]) gave a categorification of all simple modules of $\mathfrak{g}$, for $\mathfrak{g}$ being a simple Lie algebra.

- ▶ Very much in this spirit, Webster [Web13] proposed a categorification of tensor products of simple $\mathfrak{g}$-modules.

- ▶ Categorification in this spirit are a "big industry" nowadays (and it is not possible to summarize it briefly) and its very hard to overestimate the influence of the approach of Chuang–Rouquier and Khovanov–Lauda.

▷ Shortly after the groundbreaking work of (Chuang and) Rouquier, Mazorchuk–Miemietz (and their coauthors) started a systematic study of "categorifications of finite-dimensional modules of finite-dimensional algebras" (cf. [MM16a] or [MM16b] and the references therein):

- ▶ In [MM16b] they defined an appropriate 2-categorical analogue of the simple representations of finite-dimensional algebras.

- ► Several examples are know, e.g. a well-studied class of examples is given by so-called cell representations of "categorified Coxeter groups", see [MM11].
- ► At the moment, even in the case of Coxeter groups, not much is known, see e.g. [KMMZ16], [MT16] or [MMMT16].

2. Recently finished projects

This section is intended to describe some of my latest research projects.

2.1. Webs calculi in representation theory.

Paper [ST17]

A. Sartori and D. Tubbenhauer, *Webs and q-Howe dualities in types* BCD, https://arxiv.org/abs/1701.02932.

Abstract. We define web categories describing intertwiners for the orthogonal and symplectic Lie algebras, and, in the quantized setup, for certain orthogonal and symplectic coideal subalgebras. They generalize the Brauer category, and allow us to prove quantum versions of some classical type BCD Howe dualities.

2.1.1. *Diagrammatic representation theory.* Consider the following question:

Given some Lie algebra $\mathfrak{g}$, can one give a generator-relation presentation for the category of its finite-dimensional representations, or for some well-behaved subcategory?

Maybe the best-known instance of this is the case of the monoidal category generated by the vector representation of $\mathfrak{sl}_2$, or by the corresponding representation of its quantized enveloping algebra of $\mathfrak{sl}_2$. Its generator-relation presentation is known as the Temperley–Lieb category and goes back to work of Rumer–Teller–Weyl and Temperley–Lieb (the latter in the quantum setting).

2.1.2. *Web calculi and representation theory.* In pioneering work, Kuperberg [Kup96] extended this to all rank 2 Lie algebras and their quantum enveloping algebras. However, it was not clear for quite some time how to extend Kuperberg's constructions further (although some partial results were obtained). Then, in seminal work [CKM14], Cautis–Kamnitzer–Morrison gave a generator-relation presentation of the monoidal category generated by (quantum) exterior powers of the vector representation of quantum $\mathfrak{gl}_n$.

Their crucial observation was that a classical tool from representation and invariant theory, known as skew Howe duality [How95, How89], can be quantized and used as a device to describe intertwiners of quantum $\mathfrak{gl}_n$.

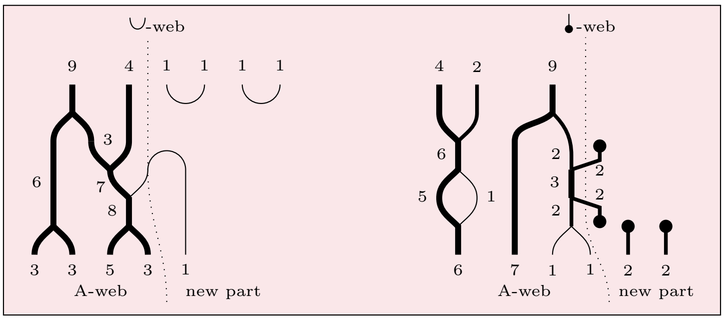
In fact, as explained in [CKM14], they allow a nice diagrammatic interpretation via so-called A-webs.

2.1.3. *Other classical groups.* The results from [CKM14] were then extended to various other instances. But, to the best of our knowledge, all generalizations so far stay in type A.

The idea which started the paper [ST17] was to extend Cautis–Kamnitzer–Morrison's approach to types BCD. However, the main obstacle immediately arises: while the quantization of skew Howe duality is fairly straightforward in type A, it is very unclear how this should be done in the other classical cases.

One of the main ideas to overcome this is to use non-monoidal web calculi. That is, our main tool are certain diagrams made out of trivalent graphs with edge labels from $\mathbb{Z}_{\geq 0}$, which we call A-, $\cup$- and ⫰-webs.

The A-webs where introduced in [CKM14] and assemble into a monoidal category. The $\cup$- and ⫰-webs were introduced in [ST17]. Let us summarize all the reader needs to know about them at the moment in a picture:



Using these, we are not just able to extend the web calculi to types BCD – generalizing Brauer's classical diagram calculus [Bra37] – but we also quantize Howe's dualities [How95, How89] for orthogonal and sympletic groups.

2.2. Link homologies and their functoriality.

Paper [ETW17]

M. Ehrig, D. Tubbenhauer and P. Wedrich, *Functoriality of colored link homologies*, https://arxiv.org/abs/1703.06691.

Abstract. We prove that the bigraded, colored Khovanov–Rozansky type A link and tangle invariants are functorial with respect to link and tangle cobordisms.

2.2.1. *Link and tangle invariants.* As already mentioned in Section 1.1.4, Khovanov [Kho00] introduced a link homology theory categorifying the Jones polynomial. His construction is one of the main examples of categorification has has attracted a lot of attention from various fields of mathematics and physics. Building on Khovanov's categorification of the Jones polynomial, Khovanov–Rozansky [KR08] introduced a link homology theory categorifying the type A Reshetikhin–Turaev invariant. Their homology theory associates bigraded vector spaces to link diagrams, two of which are isomorphic whenever the diagrams differ only by Reidemeister moves. In the original formulation, the link invariant, thus, takes values in isomorphism classes of bigraded vector spaces.

2.2.2. *Functoriality.* As mentioned in Section 1.1.4, one of the main features of Khovanov's link homology are its functorial properties. To elaborate a bit:

The first question posed by this construction is whether there is a natural choice of Reidemeister move isomorphisms, such that any isotopy of links in $\mathbb{R}^3$ gives rise to an explicit isomorphism between the Khovanov–Rozansky invariants, which only depends on the isotopy class of the isotopy. A positive answer to this question provides a functor:

$$\left\{\begin{matrix}\text{link embeddings in } \mathbb{R}^3 \\ \text{isotopies modulo isotopy}\end{matrix}\right\} \longrightarrow \left\{\begin{matrix}\text{bigraded vector spaces} \\ \text{isomorphisms}\end{matrix}\right\}$$

The second question building on the first, is whether this functor can be extended to a functor:

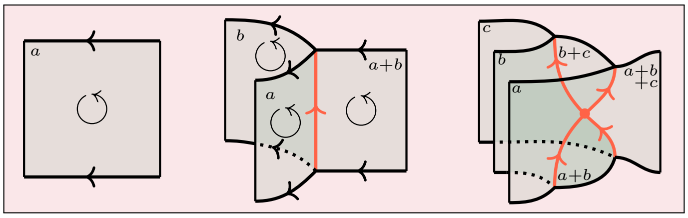
$$\left\{\begin{matrix}\text{link embeddings in } \mathbb{R}^3 \\ \text{link cobordisms in } \mathbb{R}^3 \times [0,1] \text{ modulo isotopy}\end{matrix}\right\} \to \left\{\begin{matrix}\text{bigraded vector spaces} \\ \text{homogeneous linear maps}\end{matrix}\right\}$$

2.2.3. *Functoriality of colored link homologies.* The goal of the paper [ETW17] is to answer both questions affirmatively, i.e. to prove the functoriality of Khovanov–Rozansky link homologies under link cobordisms.

We prove the general functoriality statement in a framework that is different to and more general than Khovanov–Rozansky's construction in [KR08]. For example, the so-called Murakami–Ohtsuki–Yamada state-sum model determines the Reshetikhin–Turaev invariants of links whose components are colored by fundamental representations. One categorical level up, there is an analogous extension of Khovanov–Rozansky's original *uncolored* – i.e. colored only with the vector representation – construction to the colored case. In [ETW17], we work in this generality.

Note also that we have decided to present the results of the paper [ETW17] using the ground ring $\mathbb{C}$. This is for notational convenience: with minimal adjustments our proof of functoriality also works over $\mathbb{Z}$.

Our main tool is the graphical calculus of foams, i.e. certain singular cobordisms locally modeled on



These foam models provide an accessible – and local – way to define and study the link homologies.

2.3. **Categorical representations of dihedral groups.**

Paper [MT16]

M. Mackaay and D. Tubbenhauer, *Two-color Soergel calculus and simple transitive 2-representations*, https://arxiv.org/abs/1609.00962.

Abstract. In this paper we complete the classification of simple transitive 2-representations of the small quotients of Soergel bimodules in finite dihedral type. In particular, we use bipartite graphs to give an explicit construction of a graded (non-strict) version of all these 2-representations, including the ones whose underlying graph is a Dynkin diagram of type E.

Moreover, we give two simple combinatorial criteria for when two such 2-representations are equivalent and for when their Grothendieck groups give rise to isomorphic representations.

Finally, our construction also gives a large class of simple transitive 2-representations in infinite dihedral type.

A more abstract framework is discussed in a more recent follow-up:

Paper [MMMT16]

M. Mackaay, V. Mazorchuk, V. Miemietz and D. Tubbenhauer, *Simple transitive 2-representations via (co)algebra 1-morphisms*, https://arxiv.org/abs/1612.06325.

Abstract. For any fiat 2-category $\mathscr{C}$, we show how its simple transitive 2-representations can be constructed using coalgebra 1-morphisms in the injective abelianization of $\mathscr{C}$. Dually, we show that these can also be constructed using algebra 1-morphisms in the projective abelianization of $\mathscr{C}$. We also extend Morita–Takeuchi theory to our setup and work out several examples, including that of Soergel bimodules for dihedral groups, explicitly.

2.3.1. *Motivation: "higher representation theory".* An essential problem in classical representation theory is the classification of the simple representations of any given algebra, i.e. the parametrization of their isomorphism classes and the explicit construction of a representative of each class.

In 2-representation theory, the actions of algebras on vector spaces are replaced by functorial actions of 2-categories on certain additive or abelian 2-categories. (See also Section 1.2 above.)

Examples are 2-representations of the 2-categories which categorify representations of quantum groups, due to (Chuang-)Rouquier [Rou08] and Khovanov-Lauda [KL10], and 2-representations of the 2-category of Soergel bimodules, which categorify representations of Hecke algebras of Coxeter groups.

Mazorchuk and Miemietz [MM16b] defined an appropriate 2-categorical analogue of the simple representations of finite-dimensional algebras, which they called simple transitive 2-representations (of finitary 2-categories).

Recall that representation theory of finite groups had a major impact on modern mathematics and related fields. The group ring of a finite group is a blueprint example of a finite-dimensional algebra, and, following Mazorchuk and Miemietz, it makes sense to ask whether we can understand its categorical representation theory.

This problem is completely out of reach at the moment. And even if one focuses on Coxeter groups (which are a well-behaved family of finite groups), their associated Hecke algebras and categorifications given by Soergel bimodules, the problem of the classification of their simple transitive 2-representations is very hard in general and not well understood, except in type A.

Thus, it is worthwhile to try to understand several instances of this in more details.

2.3.2. *The case of small quotients.* The authors of [KMMZ16] studied the so-called small quotient of Soergel bimodules (this basically means that one "kills higher cells") and their simple transitive 2-representations, for all finite Coxeter types. These 2-representations are given by categories on which the bimodules act by endofunctors and the bimodule maps by natural transformations. Each of these categories is equivalent to the (projective or abelian) module category over the path algebra of a finite quiver, which can be obtained by doubling a Dynkin diagram whose type depends on the 2-representation. An almost complete classification was given in [KMMZ16]:

In every finite Coxeter type of rank strictly greater than two, all the simple transitive 2-representations are equivalent to Mazorchuk and Miemietz's categorification of the cell representations of Hecke algebras [MM11], the so-called cell 2-representations.

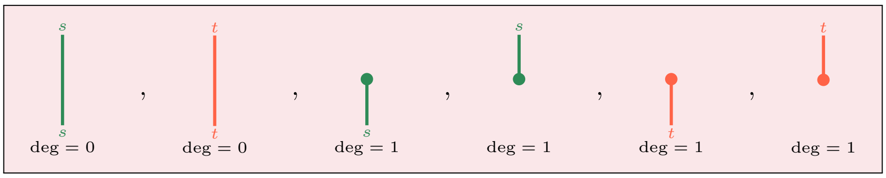
2.3.3. *Attacking rank two.* The rank two case is more delicate. In dihedral type $\mathrm{I}_2(n)$, for any odd $n \in \mathbb{Z}_{>1}$, there are two, inequivalent, cell 2-representations. For such n, these exhaust all simple transitive 2-representations, up to equivalence.

When $n = 2, 4$, it was already known that the same holds, see [Zim17]. However, when n is even and greater than four, it was shown in [KMMZ16] that there exist additional simple transitive 2-representations which are not equivalent to cell 2-representations. If one has $n \notin \{12, 18, 30\}$, then there exist exactly two of these, and all simple transitive 2-representations are either one of these or cell 2-representations.

These simple transitive 2-representations were constructed intrinsically in the paper [KMMZ16]: The cell 2-representations, due to Mazorchuk and Miemietz [MM11], can be constructed as subquotients of the 2-category of Soergel bimodules. The additional simple transitive 2-representations of $\mathrm{I}_2(n)$, for $n > 5$ even, were constructed in [KMMZ16] using an involution on the cell 2-representations, mimicking a construction in [MM17].

In some sense, a more explicit construction was needed to fill in the missing cases $n \in \{12, 18, 30\}$ and which hopefully works even more general. (Left aside the question whether such an explicit construction might be useful for future "applications".)

2.3.4. *A new approach using diagrammatics.* In [MT16], we construct all simple transitive 2-representations of the small quotient of the Soergel bimodules of type $\mathrm{I}_2(n)$ by different means: we use Elias' [Eli16] diagrammatic version of the latter 2-category, the so-called two-color Soergel calculus. Here, as an example, some of the diagrammatic generators:



More precisely, given a Dynkin diagram of type A, D or E with a bipartition, we define two self-adjoint endofunctors $\mathbf{\Theta}_s$ and $\mathbf{\Theta}_t$ (one for each generator of $\mathrm{I}_2(n)$) on the module category over the corresponding quiver, and a natural transformation between composites of them for each generating diagram in the two-color Soergel calculus, such that all diagrammatic relations are preserved.

This construction also gives graded versions of these and graded simple transitive 2-representations over the Soergel bimodules in type $\mathrm{I}_2(\infty)$ (there exists no small quotient in this case) for any finite bipartite graph.

Moreover, in finite and infinite type, we show that two graded simple transitive 2-representations are equivalent if and only if the corresponding bipartite graphs are isomorphic (as bipartite graphs). Finally, we also determine when graded simple transitive 2-representations decategorify to isomorphic representations of the corresponding Hecke algebra, using a purely graph-theoretic property.

An interesting consequence of these results is that there are two inequivalent graded simple transitive 2-representations of type E_6 (or E_8) for the Soergel bimodules of type $I_2(12)$ (or $I_2(30)$), which decategorify to isomorphic representations of the associated Hecke algebra. (The two simple transitive 2-representations of type E_7 for the type $I_2(18)$ Soergel bimodules have non-isomorphic decategorifications.) There are also inequivalent graded simple transitive 2-representations with isomorphic decategorifications over the type $I_2(\infty)$ Soergel bimodules. To the best of our knowledge these are the first examples of such 2-representations of the same 2-category which decategorify to isomorphic representations.

3. Ongoing projects

3.1. **Categorical representations.** As stated above (in e.g. Section 2.3), the subject of categorical representations is widely open at the moment. Hence, there are still a lot of "gold nuggets hidden which are not very deep under the surface".

Since I believe in the future usefulness of Mazorchuk–Miemietz's approach, I want continue my research on 2-representations of Soergel bimodules and related 2-categories.

To be a bit more precise, the following are explicit questions on which I work jointly with (a subset of) Elias, Mackaay, Mazorchuk and Miemietz.

Note that the approach taken in [MT16] is motivated by joint work with Andersen [AT17]. In [AT17] we basically connect the Soergel bimodules in dihedral type to the representation theory of quantum $\mathfrak{sl}_2$ at roots of unity by defining a 2-representation on the category of tilting modules of quantum $\mathfrak{sl}_2$. Thus, one can expect a direct connection of our work to the work of Kirillov–Ostrik [KJO02] on "finite subgroups" of quantum $\mathfrak{sl}_2$ at roots of unity and the quantum analogue of the McKay correspondence (with potential connection to modular invariants in conformal field theory). This connection is made rigorous in joint work with Mackaay–Mazorchuk–Miemietz [MMMT16]. Morally: "finite subgroups" of quantum $\mathfrak{sl}_2$ at roots of unity are "fusion subquotients" of the 2-category of singular Soergel bimodules of dihedral type, and a generalization to higher ranks should work.

Moreover, since the ideas from [AT17] heavily draw from a construction of Khovanov–Seidel [KS02] (which is related to categorical actions of braid groups), one might expect connections to braid groups as well.

Another path we are exploring at the moment: We hope that the construction presented in [MT16] generalizes to other (infinite) Coxeter types, and, using more general graphs, we expect a similar construction to give 2-representations over Soergel bimodules of finite Coxeter type with higher order apex (i.e. which do not descend to the small quotient).

3.2. **A generalization of cellularity.** In groundbreaking work, Graham–Lehrer [GL96] introduced a nowadays classical concept to representation theory, the so-called cellular algebras which come equipped with a cellular basis. The theory of cellular algebras can be seen as a device which takes an algebra and – after some linear algebra only – produces its representation theory.

A lot of algebras arising in mathematics and physics are known to be cellular. Most prominently, Temperley–Lieb and Brauer algebras (and most other diagram algebras), Hecke algebras in various flavors, arc and web algebras, cyclotomic KL-R algebras and many others. Moreover, "easy" examples such as matrix algebras or polynomial like algebras are known to be cellular.

However, it became apparent (at least for me) in joint work with Andersen–Stroppel [AST18], that there are two very different flavors of cellular basis: First, the ones without any connection to the primitive idempotents – Graham–Lehrer's cellular basis of the Temperley–Lieb algebra being an example – second, the ones with built-in primitive idempotents – the basis of the Temperley–Lieb algebra from [AST18] for instance.

In ongoing joint work with Ehrig, we try to find a good notion of "interpolation" between these two extremes, while generalizing the definition of Graham–Lehrer to include several new examples of algebras – which are know to be not cellular – into a general theory.

Last, relations to diagrammatic presentations of various Lie theoretic motivated algebras and categories – cf. Section 2.1 – need to be worked out.

3.3. **Singular topological quantum field theories.** In his pioneering work, Khovanov [Kho02] introduced the so-called arc algebra. One of his main purposes was to extend his celebrated categorification of the Jones polynomial – cf. Section 1.1.4 – from links to tangles. His idea was to interpret the link homology as certain bimodules of the arc algebra.

One of Khovanov's main ideas (as developed further by Bar-Natan [BN05]) was that the arc algebras are obtained via 1+1-dimensional topological quantum fields theories (TQFTs) as originate in work of physicists and axiomatized by Atiyah–Segal in the end of the 1980ties.

These TQFTs describe the cobordisms between the links and tangles and have led to several new results in smooth four-dimensional topology, e.g. a combinatorial proof of the Milnor conjecture regarding slice torus knots by Rasmussen and purely combinatorial constructions of exotic structures in four space by Gompf–Rasmussen– cf. Section 1.1.4.

This series of results has led to several variations and generalizations of Khovanov's original formulation, utilized in a large body of work by several researchers.

For example, left aside its knot theoretical origin, the arc algebra has interesting representation theoretical, algebraic geometrical and combinatorial properties. For instance, there are relations to (cyclotomic) KL–R algebras, knot homologies, to the Alexander polynomial and knot Floer homology, to the representation theory of Brauer's centralizer algebras and to Lie superalgebras – just to name a few.

Note hereby that the formulation by Khovanov–Bar-Natan can be seen as the $\mathfrak{sl}_2$ case of a broader story, where the generalization of the usage of 1+1-dimensional TQFTs is replaced by what is called a singular 1+1-dimensional TQFTs.

Again, this point of view is fairly novel and a lot of questions remain open. For example, in joint work with Ehrig–Wilbert [ETW16], we constructed the type D version of Khovanov's arc algebra using a singular TQFT approach à la Khovanov–Bar-Natan, revealing its potential application in low-dimensional topology.

However, while the representation theoretical origin, connections and properties of the type D version of Khovanov's arc algebra are fairly well-understood – see e.g. [ES16] – its connection to low-dimensional topology remains mysterious.

It is ongoing joint work with Stroppel–Wilbert to connect the singular TQFT version of Khovanov's type D arc algebra to invariants of links and tangles – making a connection from Lie theory to low-dimensional topology.

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D.T.: Mathematisches Institut, Universität Bonn, Endenicher Allee 60, Room 1.003, 53115 Bonn, Germany

E-mail address: dtubben@math.uni-bonn.de