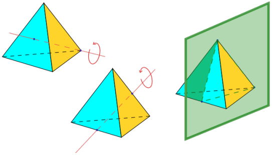
What is...a group?

Or: Abstract symmetries

Two incarnations of the same beast

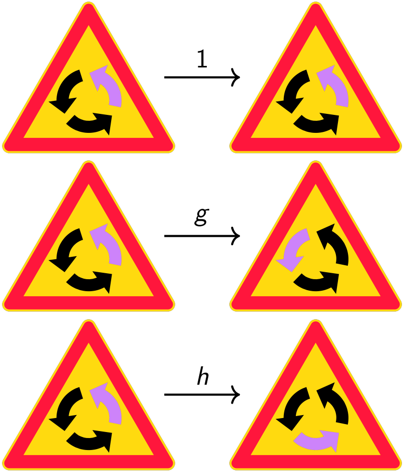
	Abstract	Incarnation
Numbers	3	 or…
Groups	$S_4 = \langle s, t, u \mid \text{some relations} \rangle$	 or…

Abstract groups formalize the concept of symmetry

Two incarnations of cyclic groups a.k.a. rotational symmetries

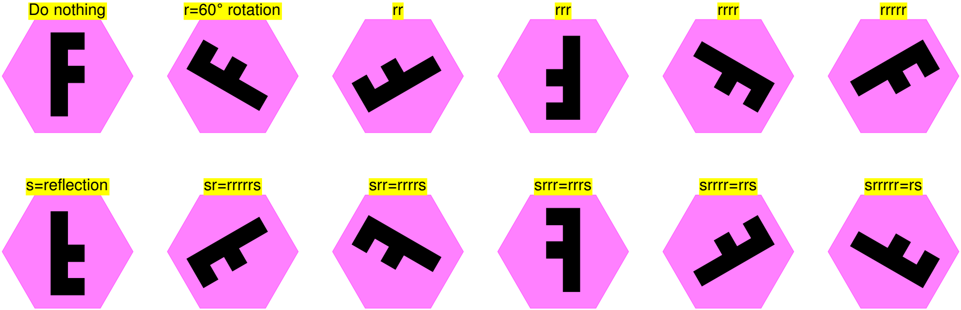
$\cdot$	1	g	h
1	1	g	h
g	g	h	1
h	h	1	g

e.g. $gh = 1$



What symmetries satisfy

- We have a composition rule $\circ(g, h) = gh$ Multiplication
- We have $g(hf) = (gh)f$ Associativity
- There is a do nothing operation $1g = g = g1$ Unit
- There is an undo operation $gg^{-1} = 1 = g^{-1}g$ Inverse



For completeness: A formal definition

A group G is a set together with a map

$$\circ\colon G \times G \to G, \circ(g, h) = gh$$ composition

such that:

(a) $\circ$ is associative: $g(hf) = (gh)f$ Associativity

(b) There exists $1 \in G$ such that $1g = g = g1$ Unit

(c) For all $g \in G$ there exists g^{-1} such that $gg^{-1} = 1 = g^{-1}g$ Inverse

Examples.

- Symmetry groups of "things" with $\circ$ =composition
- Symmetric groups S_n, alternating groups A_n
- Cyclic groups $\mathbb{Z}/n\mathbb{Z}$ with $\circ$ =addition
- $\mathbb{Z}$ with $\circ$ =addition
- $\mathbb{Q} \setminus \{0\}$ with $\circ$ =multiplications

Symmetry groups of the platonic solids

	Without reflections	With reflections
Tetrahedron	A_4 of order 12	S_4 of order 24
Cube+Octahedron	S_4 of order 24	$S_4 \times \mathbb{Z}/2\mathbb{Z}$ of order 48
Dodecahedron+Icosahedron	A_5 of order 60	$A_5 \times \mathbb{Z}/2\mathbb{Z}$ of order 120

Thank you for your attention!

I hope that was of some help.