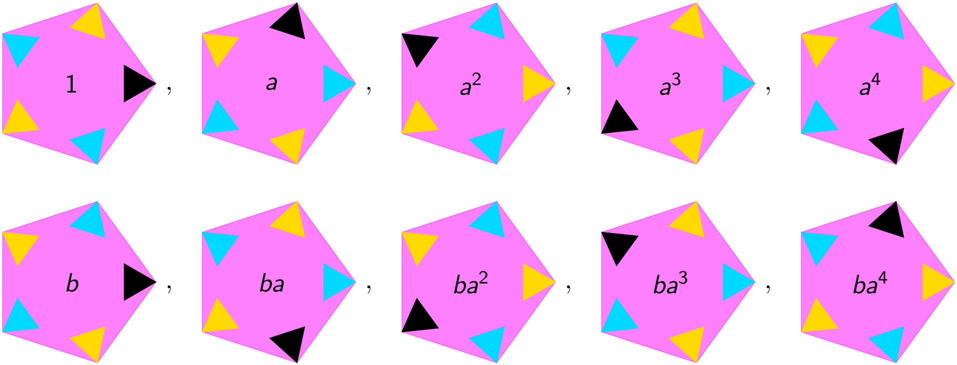
What are...the Sylow theorems?

Or: Canonical substructures?

The symmetry group D_{10} of the pentagon

The generating symmetries of a pentagon are a rotation a and a reflection b

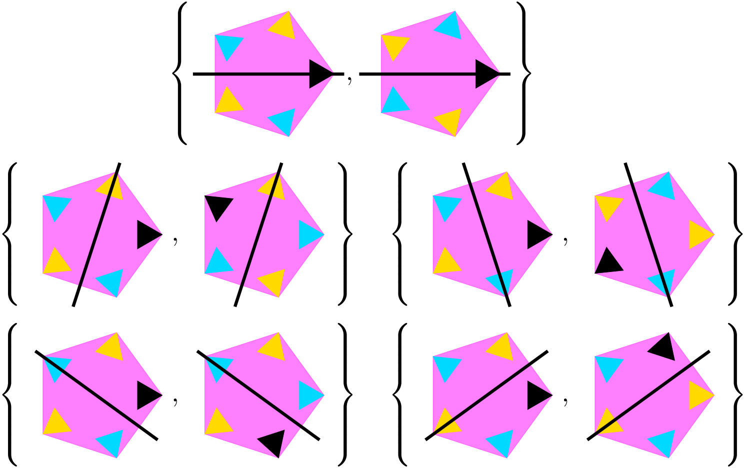


D_{10} is of order $|D_{10}| = 10 = 2 \cdot 5$

Subgroups of order 2?

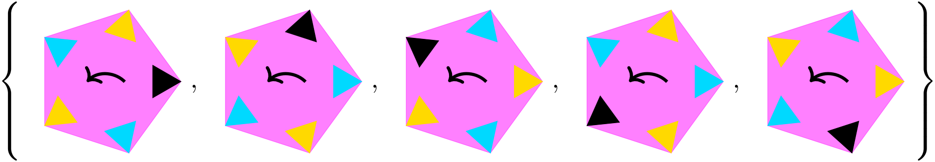
There are five subgroups of order 2

All of them are conjugate by rotation



Subgroups of order 5?

There is one subgroup of order 5
So it is unique



For completeness: The formal statements

G a finite group, $|G| = p^r m = p^r m$ with p prime, $p \not\div m$. Then:

(a) For all $1 \leq s \leq r$ there exists a subgroup of order p^s

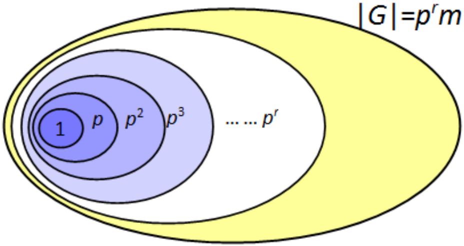
(b) A subgroup of order p^r is called a p Sylow subgroup

(c) All p Sylow subgroups are conjugate Uniqueness

(d) The number n_p of p Sylow subgroups satisfies $n_p \div m$ and $p \div (n_p - 1)$

(e) A p Sylow subgroup is normal if and only if $n_p = 1$

The p Sylow subgroup is the (up to conjugation) unique end of a tower:



Uniqueness might looks innocent, but...

The symmetric group has a lot of non-conjugate subgroups:

	Order	Number of subgroups	Number up to conjugacy
S_1	1	1	1
S_2	2	2	2
S_3	$6 = 2 \cdot 3$	6	4
S_4	$24 = 2^2 \cdot 3$	30	11
S_5	$120 = 2^2 \cdot 3 \cdot 5$	156	19
S_6	$720 = 2^3 \cdot 3^2 \cdot 5$	1455	56
S_7	$5040 = 2^3 \cdot 3^2 \cdot 5 \cdot 7$	11300	96
S_8	$40320 = 2^7 \cdot 3^2 \cdot 5 \cdot 7$	151221	296

The uniqueness of the p Sylow subgroups is a very strong property

Thank you for your attention!

I hope that was of some help.