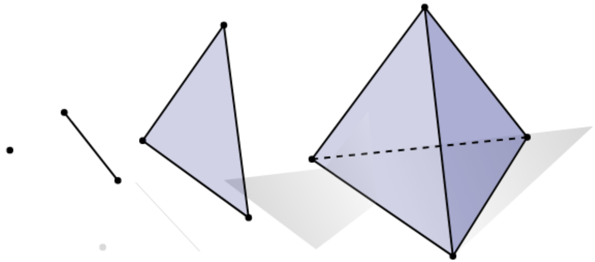
What are...simplicial complexes?

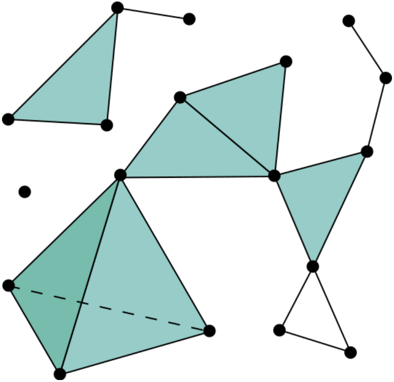
Or: Triangles everywhere

Lots of triangles



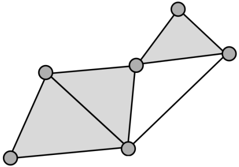
- A 0 dimensional triangle is a point
- A 1 dimensional triangle is a line
- A 2 dimensional triangle is a solid triangle
- A 3 dimensional triangle is a solid tetrahedron
- An n dimensional triangle is called an n simplex

Lots of triangles glued together

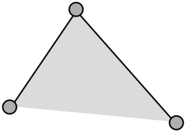


- The boundary of an n simplex is made of $n-1$ simplex
- Gluing simplices along their boundary gives simplicial complexes

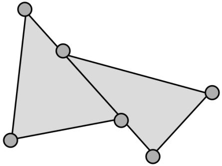
Not everything is allowed



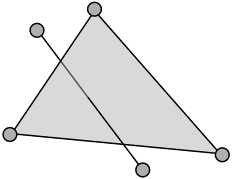
(a) A simplicial complex



(b) Missing edge



(c) Shared partial edge

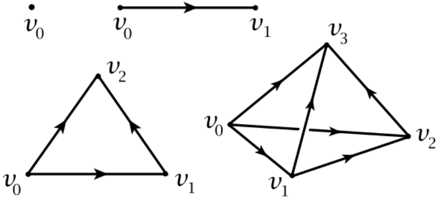


(d) Nonface intersection

(a) is ok but (b), (c) and (d) are not allowed

For completeness: A formal definition

An n simplex for $v_0, ..., v_n$ is smallest convex set in $\mathbb{R}^{n+1}$ containing $v_0, ..., v_n$ that do not lie in a hyperplane of dimension less than n



If we delete one of the $n+1$ vertices of an n simplex, then the remaining n vertices span an $(n-1)$ simplex, called a face 3d terminology

Sometimes one needs an orientation, but this is ignored in this video

A simplicial complex Δ is a set of simplices satisfying

- Every face of a simplex from Δ is also in Δ (b) from before
- A $\neq \emptyset$ intersection of 2 simplices in Δ is a face of both (c)+(d) from before

Abstract simplicial complexes

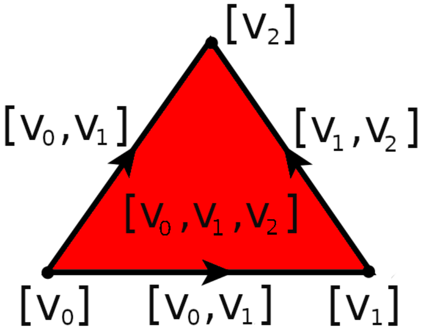
- Abstract simplicial complexes Δ_{ab} are collections of non-empty sets satisfying

$$(X \in \Delta_{ab} \text{ and } Y \subset X) \Rightarrow (Y \in \Delta_{ab})$$

These can be geometrically realized into simplicial complexes

- Example The abstract standard 2-simplex and its geometric realization:

$$\Delta^2_{ab} = \left\{ \begin{matrix} [v_0], [v_1], [v_2], [v_0, v_1], \\ [v_0, v_2], [v_1, v_2], [v_0, v_1, v_2] \end{matrix} \right\} \leftrightsquigarrow$$



Thank you for your attention!

I hope that was of some help.