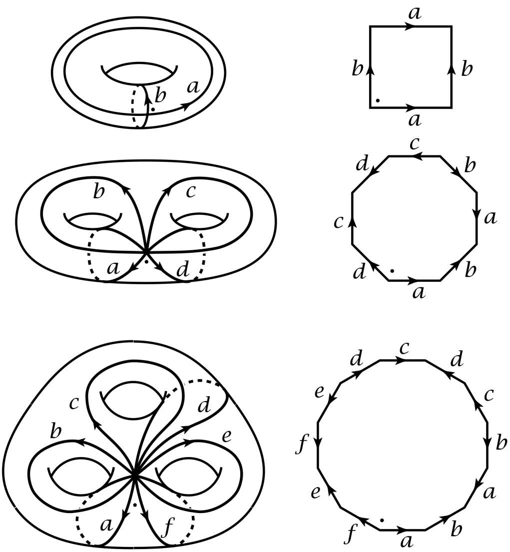
What are...cell complexes?

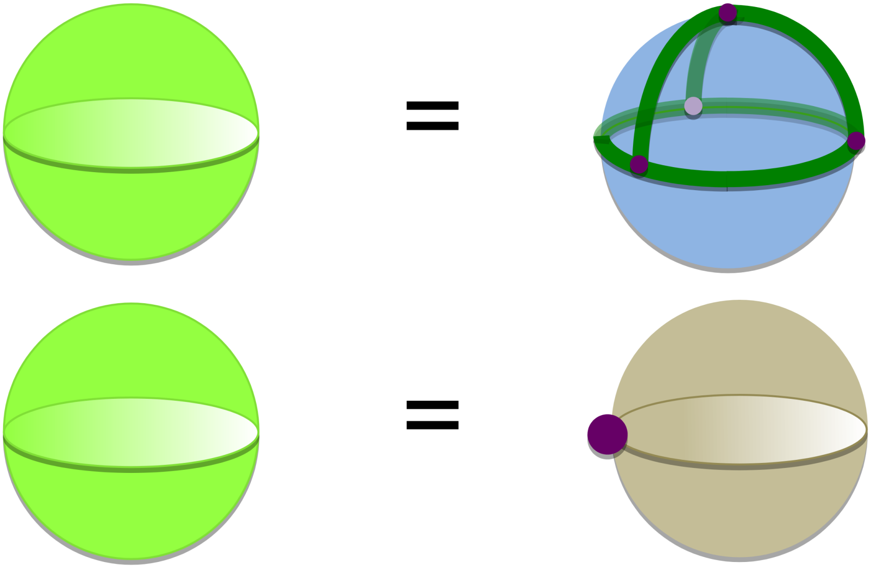
Or: Constructed from discs

From polygons to donuts



These are constructed from discs/cells: $D^0 =$ point, $D^1 =$ interval $D^2 =$ disc

Two ways to construct a sphere



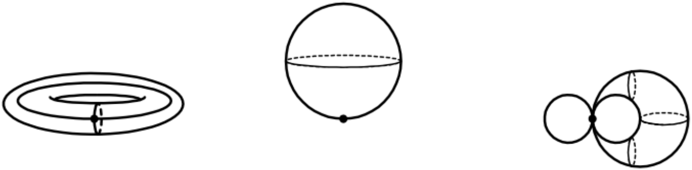
Warning. A space can have many cell structures – or none at all!

A construction recipe

- Start with a set of points **Add 0-cells**

- Glue lines to the points along their boundary **Add 1-cells**

- Glue discs to the lines along their boundary **Add 2-cells**



- Continue in this way **Add *n*-cells**

For completeness: A formal definition

A cell complex X is constructed inductively via a cell structure:

(a) Start with a discrete set X^0 of points, the 0-cells

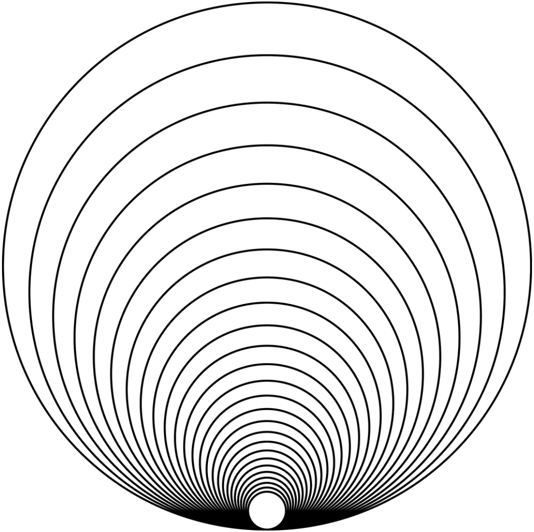
(b) Form X^n from X^{n-1} by attaching n-cells via maps $\phi_\alpha \colon S^{n-1} \to X^{n-1}$

(c) This means X^n is the quotient of $X_{n-1} \coprod_\alpha D^n_\alpha$ under the identification given by ϕ_α

(d) A subset of X is closed if and only if it meets the closure of each cell in a closed set

- Having a cell structure gives tools to compute various constructions in algebraic topology Cool!
- Such X are also known as CW complexes
- The spaces $X^0 \subset X^1 \subset ...$ are the n-skeletons
- There are various finiteness conditions one could impose such as the number of cells is finite
- A topological space might have many cell structures or none at all

A non-example



The Hawaiian earring does not admit a cell structure

This makes it hard to compute *e.g.* the fundamental group

Thank you for your attention!

I hope that was of some help.