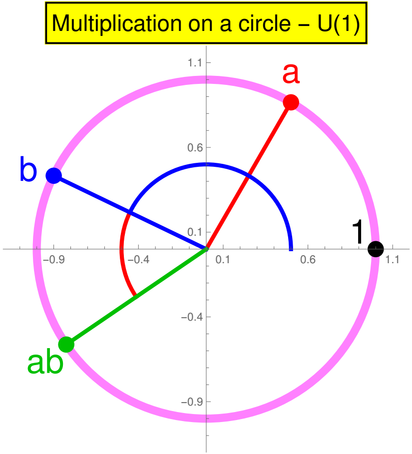
What is...the smooth periodic table?

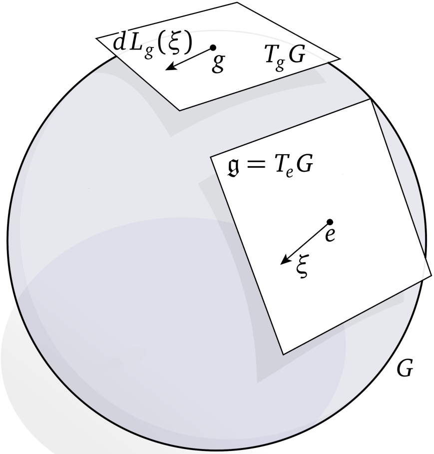
Or: Simple Lie groups

The best of both worlds



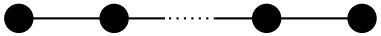
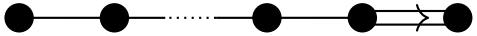
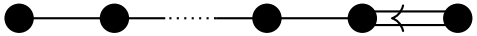
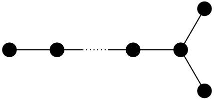
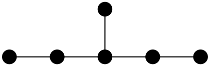
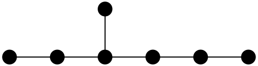
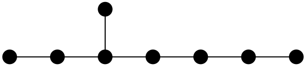
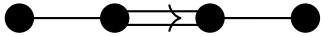
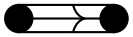
- A circle is an object of (differential) geometry Smooth manifold
- A circle is an object of algebra Group

A Lie group - fundamental for both worlds



A Lie group = smooth manifold + smooth group structure

The periodic table – the simplest Lie groups

Name	Diagram	Lie algebra	Lie group
A_n		$\mathfrak{sl}_n(\mathbb{C})$	$\mathrm{SL}_n(\mathbb{C})$
B_n		$\mathfrak{so}_{2n+1}(\mathbb{C})$	$\mathrm{SO}_{2n+1}(\mathbb{C})$
C_n		$\mathfrak{sp}_{2n}(\mathbb{C})$	$\mathrm{SP}_{2n}(\mathbb{C})$
D_n		$\mathfrak{so}_{2n}(\mathbb{C})$	$\mathrm{SO}_{2n}(\mathbb{C})$
E_6		Exceptional	Exceptional
E_7		Exceptional	Exceptional
E_8		Exceptional	Exceptional
F_4		Exceptional	Exceptional
G_2		Exceptional	Exceptional

Enter, the theorem!

A centerless connected complex Lie group is called simple if its Lie algebra is simple.

(No universally accepted definition and I take one of them.)

They are classified as:

(a) Classical types *ABCD* The matrix groups

(b) Exceptional types *EFG* A handful of exceptions

- This is not quite what I showed you because of the centerless condition
- Via coloring of the diagrams one can include the real versions as well
- All connected Lie groups arise from $\mathbb{R}$, $U(1)$ and the *ABCDEFG* types via group extensions Elementary smooth symmetries

Galois vs. Lie – discrete vs. smooth symmetries

Galois ∼1830:

Finite groups are symmetries of polynomial equations

Lie ∼1870:

Lie groups are symmetries of differential equations

Finite groups. Easy to define, hard to classify

Lie groups. Hard to define, easy to classify

Thank you for your attention!

I hope that was of some help.