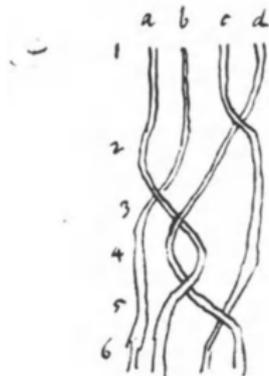


What are...Artin braid groups?

Or: Topology *via* algebra

Braids in mathematics?

Braids are around for millennia, but how to study them mathematically?



Veränderung der Anordnung

2	1	1	2+i	3+i	2+2i	2+2i
3	2	2	1	1	1	1
4	3	4	4	4	4	3+i
2	4	3+i	3+i	2+2i	3+2i	4+3i

Es kommt daraus den „Begriff“ der „Verkettung“,
 als „Hypothese“ im „Knoten“ vorzustellen, dass
 man nicht welche Teile einander bestimmen.

Wahrscheinlich wird es erreichen die halben „Wahrscheinlichkeiten“
 einer Linie um die andere nach einem bestimmten Drehungs-
 Winkel angegeben.
 In diesem Beispiel
 cd, ab, cd, ab
 cd, ab

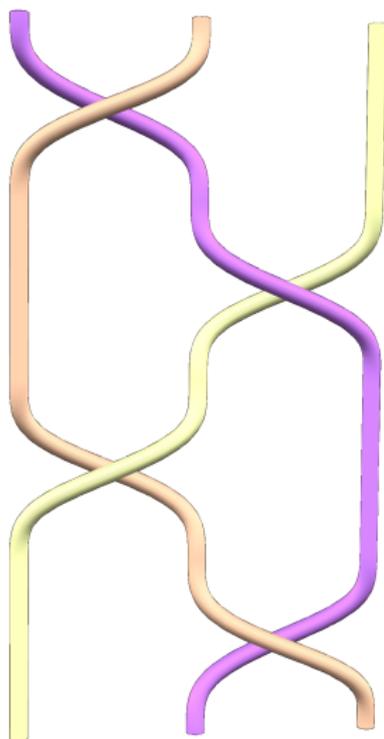


Man braucht nur in jeder Linie zu zählen wie oft + mit - wechselt

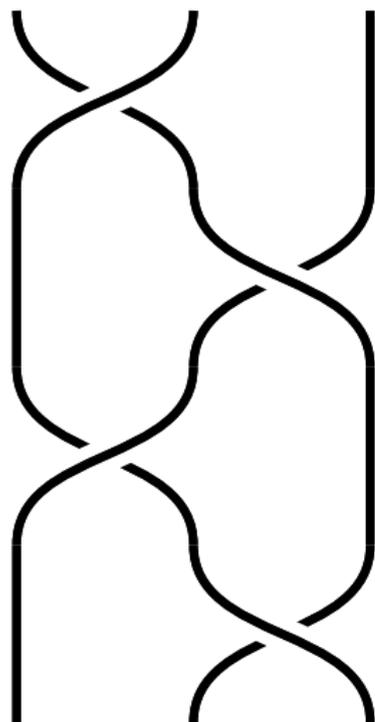
Gauss' handwritten notes ~1820: first appearance of braids in mathematics?

Braids in 2d

$3d$:



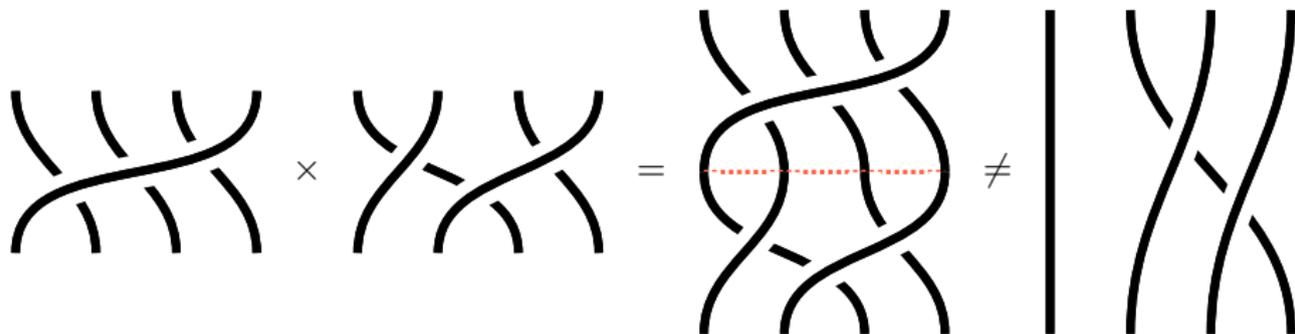
$2d$:



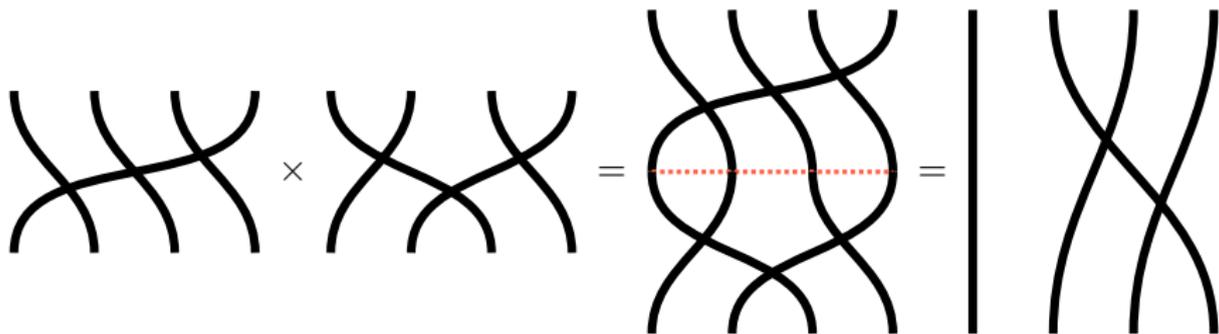
Question. Can one describe the information **loss** from 3d to 2d?

Its a group! gh is "stack g on top of h "

Braid group (infinite unless you just have one strand):



Symmetric group (always finite):



Enter, the theorem!

(a) Braids (topology) on n strands form a group Br_n (algebra)

(b) The group Br_n is generated by



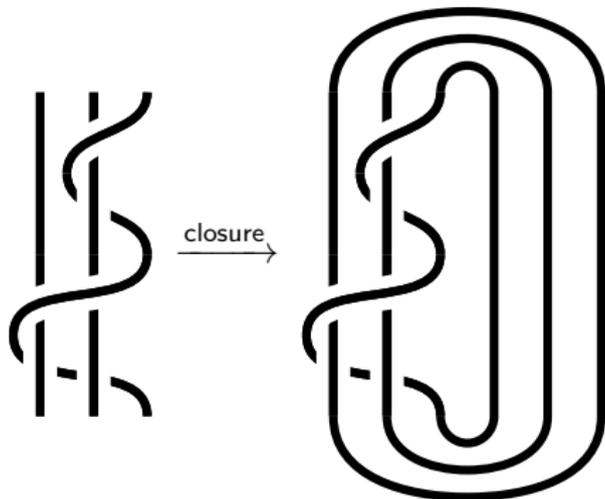
(c) Two elements in Br_n represent the same braid if and only if they are related by height moves or



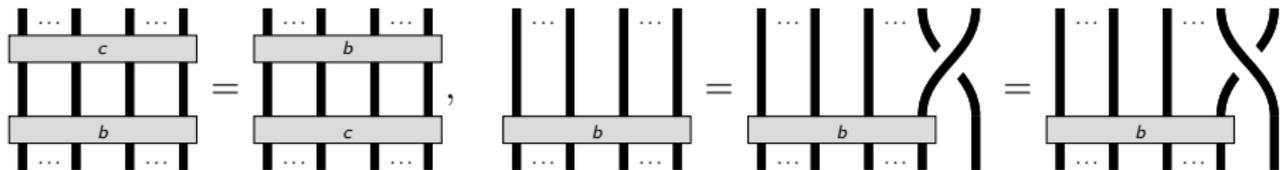
Consequences.

- ▶ One gets a purely algebraic way to study braids
- ▶ The symmetric group is a quotient, so one also gets a presentation for it

A purely algebraic way to study knots/links



- ▶ Alexander theorem. Every knot/link arises in this way
- ▶ Markov theorem. Two closures represent the same knot/link if and only if they are related by braid operations or



Thank you for your attention!

I hope that was of some help.