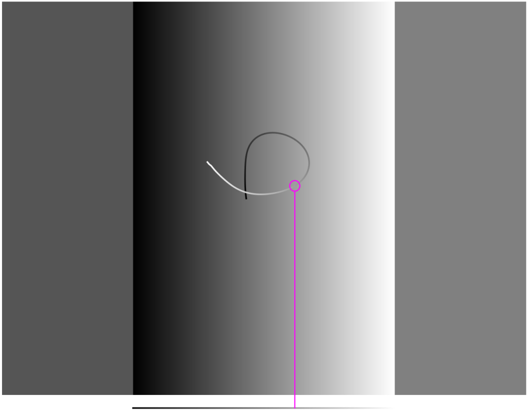
What is...the Brouwer fixed point theorem?

Or: Why lines need to cross.

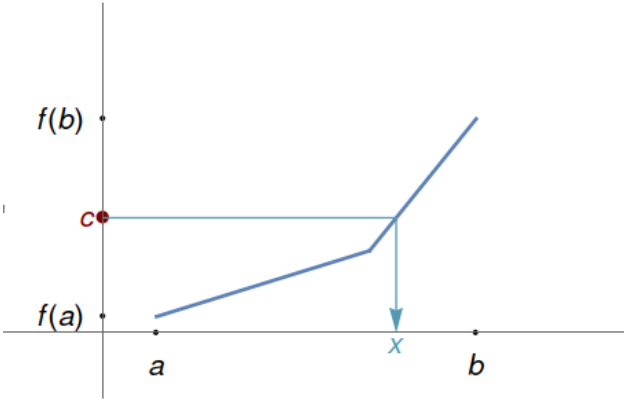
Put a colored string into a background of the same color code:



Fun observation. At least one point is fixed, *i.e.* of the same color as its background

The intermediate value theorem

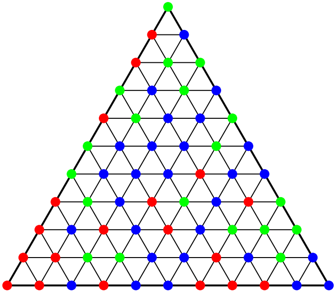
If $f : [a, b] \to [f(a), f(b)]$, and $c \in [f(a), f(b)]$, then there is at least one $x \in [f(a), f(b)]$ such that $f(x) = c$



Wait: This c is a fixed point, namely of $f(x) - x + c$

Sperner's lemma

Crossing only red-green edges, you always get stuck in a tricolored triangle



The rule to create these: The outside triangle has three colors, the edges on the boundary the colors of the outside points – the middle is arbitrary.

A fixed point, namely of $f(x) - x$ (by repeating the process for the end triangle)

Enter, the theorem!

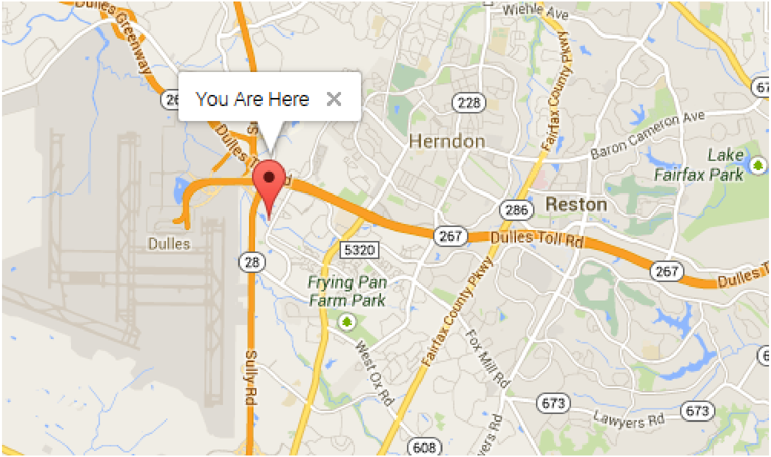
Any continuous function f sending a compact convex set onto itself contains at least one fixed point.

Compact convex sets:

- An interval
- A disk
- A triangle
- A ball
- A filled cube
- ...

From a 2d disk into a 2d disk

Brouwer's fixed point on a map: the "You are here" marker



Why? Well, a map is a disk mapped into a disk, its location

Thank you for your attention!

I hope that was of some help.