

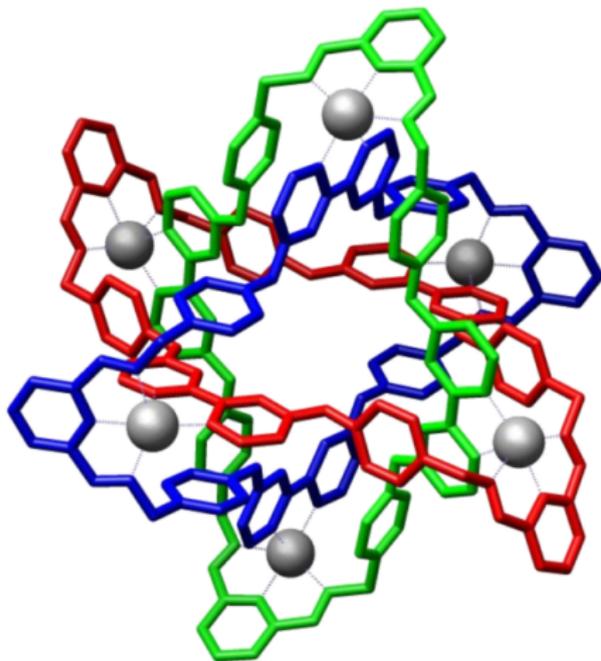
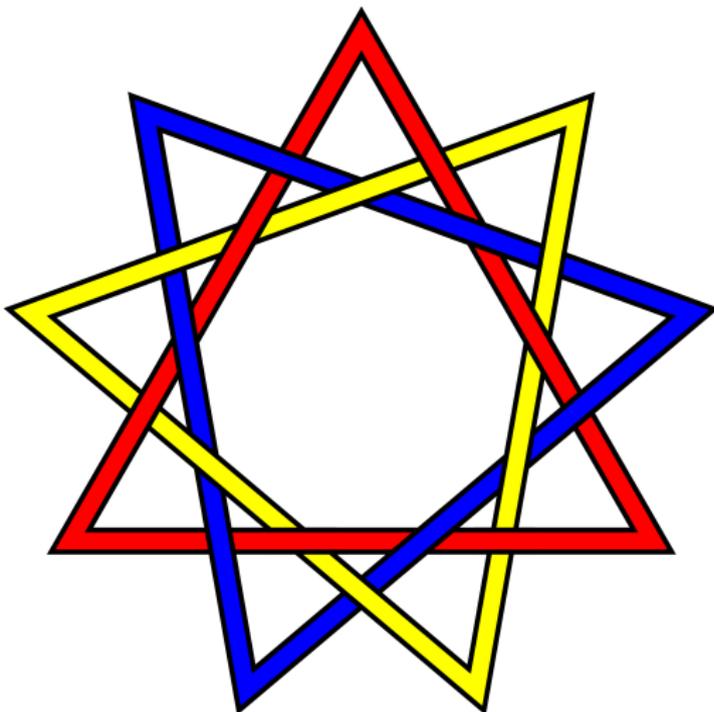
**What is...the Borromean impossibility theorem?**

---

Or: Borromean rings do not exist!

## Borromean rings in culture and nature

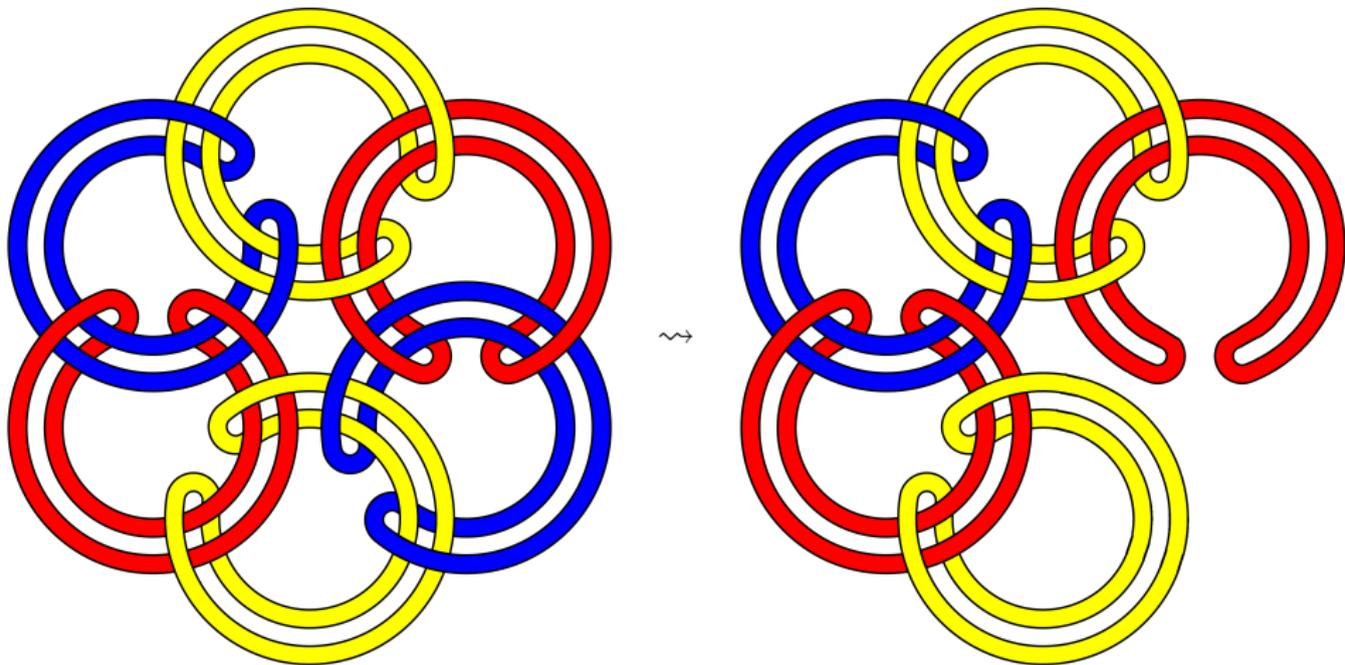
---



If you remove either of the three, then the whole things falls apart

## Brunnian links

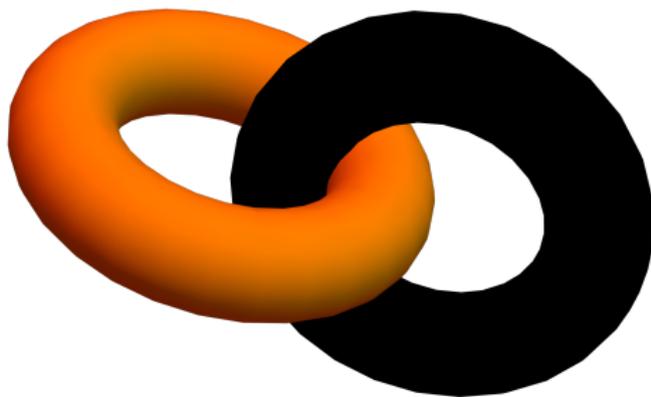
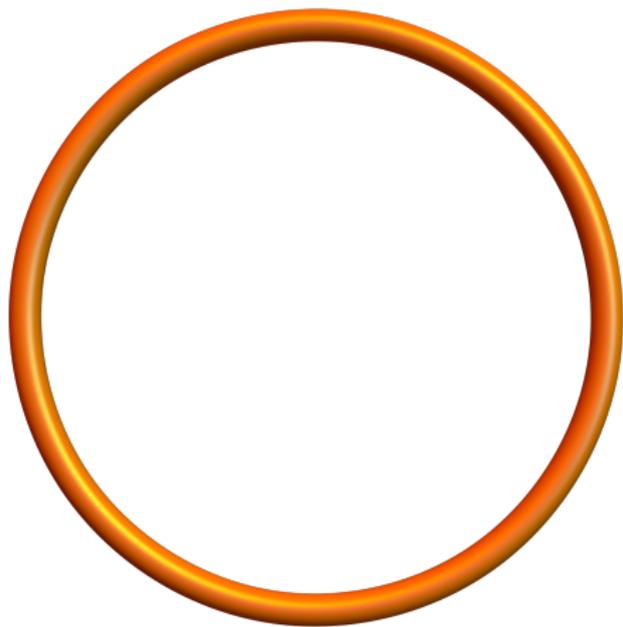
---



Borromean rings with any number of components  $n$

Are these perfect circles?

---



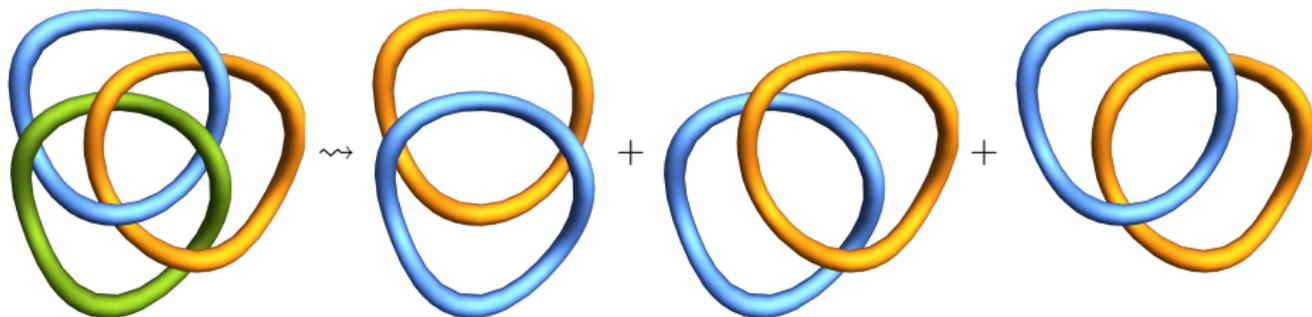
This works for  $n = 1$  unknot or  $n = 2$  Hopf link, but for  $n > 2$ ?

## Enter, the theorem

---

A link consisting of  $n > 2$  disjoint perfect circles is **trivial**

- ▶ In other words, Borromean/Brunnian links **can not** be made out of perfect circles unless  $n = 1, 2$
- ▶ Illustrations usually **cheat**, e.g.



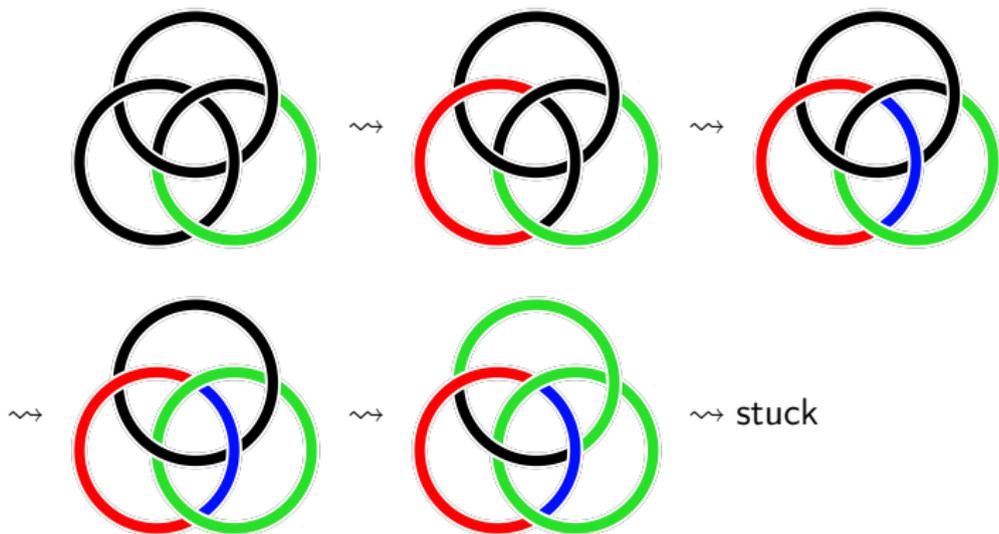
- ▶ This theorem goes back to Freedman–Skora  $\sim$  1987
- ▶ What is actually missing in the above claim is that Borromean/Brunnian links are **non-trivial** – left to the reader as an exercise ;-)

# The Borromean rings are non-trivial

- ▶ 3-colorings of crossings in diagrams:



- ▶ The number of 3-colorings is an invariant of links
- ▶ Borromean rings have **no non-trivial 3-colorings**:



**Thank you for your attention!**

---

I hope that was of some help.