

TITLES AND ABSTRACTS FOR THE CONFERENCE ‘INTERACTIONS OF LOW-DIMENSIONAL TOPOLOGY AND “HIGHER” REPRESENTATION THEORY’

LECTURES

► Michael Ehrig

- ▷ **Title.** Arc algebras: a bridge between representation theory, geometry and topology
- ▷ **Abstract.** In this series of talks we will discuss different versions of arc algebras. From the original definition of Khovanov to different generalizations and modifications. We will discuss their combinatorics and how they are related to different parts of representation theory, topology and algebraic geometry.

► Martina Lanini

- ▷ **Title.** Modular representation theory, alcove combinatorics and a category of sheaves
- ▷ **Abstract.** This lecture series is meant to provide a quick introduction to the representation theory of reductive algebraic groups in positive characteristic and to the alcove combinatorics, and then to discuss their interplay. In the last lecture a third character will make its entrance into the play: a certain category of sheaves on the alcoves, defined recently in joint work with Peter Fiebig, which provides new tools to investigate modular representations.

► Vanessa Miemietz

- ▷ **Title.** Introduction to the 2-representation theory of finitary 2-categories
- ▷ **Abstract.** In the first talk, I will give a general introduction to 2-categories in general and finitary 2-categories in particular, and explain several important examples. In the second talk, I will speak about finitary 2-representation, and explain simple transitive and cell 2-representations. In the final talk, I will introduce coalgebra 1-morphisms and explain their importance in the construction of finitary 2-representations.

► Krzysztof Putyra

- ▷ **Title.** A crash course on $\mathfrak{gl}(2)$ link homology
- ▷ **Abstract.** The introduction of $\mathfrak{gl}(2)$ link homology by Khovanov revolutionized modern knot theory. Not only led it to invariants stronger than the quantum polynomials (Khovanov homology detects unlinks), but also - due to its functorial nature - opened the world of $4D$ topology to purely combinatorial methods.
We will begin with the construction of Khovanov homology for links in thickened surfaces using dotted cobordisms introduced by Bar-Natan and TQFT functors constructed by Asaeda, Przytycki, and Sikora. Functoriality will be discussed in the second talk, in which we will replace cobordisms with $\mathfrak{gl}(2)$ foams à la Blanchet. The last talk will present our latest results on quantization of $\mathfrak{gl}(2)$ link homology and its application to the categorification of the colored Jones polynomial.

TALKS

► Christian **Blanchet**

- ▷ **Title.** Symmetrised integral on Hopf algebras and logarithmic Hennings invariants
- ▷ **Abstract.** Witten–Reshetikhin–Turaev quantum invariants use only partially the representation category of quantum $\mathfrak{sl}(2)$ at roots of unity. Hennings invariants of 3-manifolds are constructed within the quantum group itself. Unfortunately they often vanish and do not extend to full TQFT. Modified trace theory allows construction of invariants using non trivially the non semisimple structure.

We show that for finite dimensional unimodular Hopf algebra, modified trace is obtained from (right/left) integral.

(Joined work with Anna Beliakova and Azat Gainutdinov.)

We discuss the invariants which are obtained from modified trace in case of restricted quantum $\mathfrak{sl}(2)$.

(Joined work with Anna Beliakova and Nathan Geer.)

► Hankyung **Ko**

- ▷ **Title.** Quantum polynomial functors
- ▷ **Abstract.** The (strict) polynomial functors form a category equivalent to the polynomial representations for general linear groups. This functor interpretation provides a useful way to understand $GL(n)$ representation theory. I will present some generalizations of the polynomial functors, joint with Valentin Buciumas, namely the quantum polynomial functors and (quantum) ‘type BCD ’ analogues of the polynomial functors. Two important insight from the classical polynomial functors are the stability property and the composition of functors. I will describe these ideas and explain what generalizes and what does not generalize.

► Jonathan **Kujawa**

- ▷ **Title.** Modified traces in pivotal categories
- ▷ **Abstract.** The trace of a function and the dimension of an object are valuable tools in a wide variety of contexts. For example, they are used to define characters of finite group representations and to construct knot invariants. Unfortunately, they are also often zero exactly in the settings where you would like to be able to use them. I’ll explain the background, give examples, and talk about some recent joint work with Nathan Geer and Bertrand Patureau-Mirand to define replacements for the trace. These “modified traces” will have nice trace-like properties and often be non-trivial when the ordinary trace is zero.

► Andrew **Lobb**

- ▷ **Title.** Extra gradings on knot cohomology of symmetric links
- ▷ **Abstract.** If you draw the correct picture of the Figure Eight knot, you’ll see that it can be put into a position so that it has three symmetries - rotations by 180 degrees around the x , y , z axes of 3-dimensional space. These rotations generate the Klein 4-group. Two of the axes intersect the knot in two points, while the third lies disjoint from the knot.

In general, we'll see that given a symmetry on a link (the link may intersect the axis of symmetry multiple times) we induce a third grading on a version of Khovanov cohomology. This grading distinguishes, for example, between the three symmetries of the Figure Eight knot given above, and gives extra gradings on the usual annular cohomology.

Joint work with Liam Watson.

► **Marco Mackaay**

▷ **Title.** The 2-representation theory of di- and trihedral Soergel bimodules

▷ **Abstract.** I will briefly recall the correspondence between the simple transitive 2-representations of Soergel bimodules of dihedral type and the zigzag algebras associated to Dynkin diagrams of type ADE . As the dihedral groups are quotients of the Weyl group of affine type A_1 , we call this the rank one case.

The main topic of my talk is the generalization for rank two, i.e. trihedral zigzag algebras associated to generalized Dynkin diagrams of type ADE and the 2-representation theory of trihedral Soergel bimodules.

This is joint work with Mazorchuk, Miemietz and Tubbenhauer.

► **Volodymyr Mazorchuk**

▷ **Title.** Simple transitive 2-representations of Soergel bimodules

▷ **Abstract.** In this talk I plan to present what is known and what is not known about classification of simple transitive 2-representations of the 2-category of Soergel bimodules associated to a reflection faithful representation of a Coxeter group.

► **Louis-Hadrien Robert**

▷ **Title.** Foam evaluation and Kronheimer–Mrowka Theory

▷ **Abstract.** (Joint with Mikhail Khovanov.) Kronheimer and Mrowka use $SO(3)$ gauge theory to associate with every trivalent graph K embedded in a 3-manifold M a vector space $J^\sharp(K)$. With help of Gabai's sutured theory, they prove that if K is in $\mathbb{R}^2 \subset \mathbb{R}^3$ and bridgeless, then $J^\sharp(K)$ is not trivial. They conjecture that in these case the dimension $J^\sharp(K)$ equals the number of Tait coloring of K . Their conjecture implies the 4 color theorem. I will explain how to use foam evaluation to construct what appears to be combinatorial analogue of $J^\sharp(K)$.

► **David Rose**

▷ **Title.** \mathfrak{gl}_n homologies, annular evaluation, and symmetric webs

▷ **Abstract.** A conjecture of Dunfield–Gukov–Rasmussen predicts a family of differentials on reduced HOMFLYPT homology, indexed by the integers, that give rise to a corresponding family of reduced link homologies. We'll discuss the proof of a variant of this conjecture, constructing an unreduced link homology theory categorifying the quantum \mathfrak{gl}_n link invariant for all non-zero values of n (including negative values!). To do so, we employ the technique of annular evaluation, which uses categorical traces to define and characterize type A link homology theories in terms of simple data assigned to the unknot. Of particular interest is the case of negative n , which gives a categorification of the “symmetric webs” presentation of the type A Reshetikhin–Turaev

invariant, and which produces novel categorifications thereof (i.e. distinct from the Khovanov–Rozansky theory).

► Anne-Laure **Thiel**

- ▷ **Title.** On some generalizations of the category of Soergel bimodules
- ▷ **Abstract.** The category of Soergel bimodules plays an essential role in (higher) representation theory and for the construction of homological invariants in knot theory. The aim of this talk is to present a generalization of Soergel category attached to a Coxeter group of type A_2 . While the Soergel category counts a generating bimodule per simple reflection, this generalization is obtained by taking one generator per reflection. I will give a complete description of this category through a classification of its indecomposable objects and study its split Grothendieck ring. This gives rise to an algebra which is a quotient of the corresponding affine Hecke algebra of type A_2 , that can be presented by generators and relations. If time permits, I will also sketch the construction of a category which is defined in a similar way as Soergel’s one and attached to a complex reflection group of rank one.
This is joint work with Thomas Gobet.

► Pedro **Vaz**

- ▷ **Title.** DG -enhanced cyclotomic KLR algebras and categorification of Verma modules
- ▷ **Abstract.** In this talk I will present DG -enhanced versions of cyclotomic Khovanov–Lauda–Rouquier algebras and explain how to use them to categorify parabolic Verma modules for (symmetrizable) quantum Kac–Moody algebras.

► Emmanuel **Wagner**

- ▷ **Title.** Foams and categorification
- ▷ **Abstract.** (Joint work with L.-H. Robert.) I will discuss various constructions of different type A link homologies using an evaluation formula for closed foams and trivalent TQFT’s.

► Paul **Wedrich**

- ▷ **Title.** On categorification of skein modules and algebras
- ▷ **Abstract.** Khovanov homology and its cousins are usually defined as functorial invariants of links in \mathbb{R}^3 . Embracing their reliance on link projections as a virtue, they admit at least two different types of extensions to links in thickened surfaces. Building on Krzysztof Putyra’s crash course on $\mathfrak{gl}(2)$ link homology, I will explain how embeddings of surfaces give rise to spectral sequences between the surface link homologies of Asaeda, Przytycki and Sikora. I will also introduce a second kind of surface link homologies, which categorify surface skein modules and, conjecturally, their algebra structures.
This is joint work with Hoel Queffelec.

► Arik **Wilbert**

- ▷ **Title.** Exotic Springer fibers and two-boundary Temperley–Lieb algebras
- ▷ **Abstract.** We study the geometry and topology of a certain family of exotic Springer fibers from an explicit, diagrammatic point of view. These algebraic varieties appear

as the fibers under a resolution of singularities of the exotic nilpotent cone which plays a prominent role in Kato’s Deligne–Langlands type classification of simple modules for multiparameter Hecke algebras of type C . We describe our results in terms of the combinatorics of the two-boundary Temperley–Lieb algebra. This provides the general framework to construct geometric versions of Khovanov’s arc algebra arising from exotic Springer fibers.

This is joint work with Neil Saunders.

► Xiaoting **Zhang**

▷ **Title.** Analogues of centraliser subalgebras for fiat 2-categories

▷ **Abstract.** In this talk we will present a result which reduces classification of simple transitive 2-representations of a fiat 2-category to classification of simple transitive 2-representations of its certain “much smaller” subquotient 2-categories. The result resembles the reduction of classification of simple modules over a finite semigroup to classification of simple modules over its maximal subgroup.

This is a joint work with Marco Mackaay, Volodymyr Mazorchuk and Vanessa Miemietz.

► Jieru **Zhu**

▷ **Title.** Presenting Cyclotomic Schur Algebras

▷ **Abstract.** The complex reflection group $G(r, 1, d)$ is the wreath product between the cyclic group of order r and the symmetric group on d letters. Mazorchuk–Stroppel stated a Schur–Weyl type duality, that the action of a Levi subgroup of the general linear group fully centralizes the action of $G(r, 1, d)$ on the d fold tensor of the natural representation. This centralizer is also known as the cyclotomic Schur algebra. We stated this result in the Lie algebra setting, and further gave two presentations for the cyclotomic Schur algebras, in each of the classical and quantum case. One of the presentations with idempotent generators gives rise to a categorification of the cyclotomic Schur algebras, using a quotient of the Khovanov–Lauda–Rouquier 2-category which categorifies the quantum group.

This is joint work with Jonathan Kujawa.