EXERCISES 4: LECTURE FOUNDATIONS OF MATHEMATICS

Exercise 1. Let $X = \{a, b, c\}$. List all possible equivalence relations on X.

Exercise 2. Let X be a set, and let X^X denote the set of all maps $X \to X$. Further, let $S(X)$ denote the set of all bijective maps $X \to X$. Show:

(a) If $f, g \in S(X)$, then $g \circ f$ and $f \circ g$ are also in $S(X)$.

(b) If X has at least two elements, then X^X is not commutative with the operation given by $\circ$.

(c) If X has at least three elements, then $S(X)$ is not commutative with the operation given by $\circ$.

Exercise 3. Let X, Y be sets and let $\sim_X, \sim_Y$ be equivalence relations on these sets. Moreover, let $f\colon X \to Y$ be a map such that

$$(\star): \qquad (x_1 \sim_X x_2) \Rightarrow (f(x_1) \sim_Y f(x_2)) \quad \forall x_1, x_2 \in X.$$

Show that there is a unique map $[f]$ such that

$$\begin{array}{ccc} X & \xrightarrow{\ f\ } & Y \\ {\scriptstyle p_X}\downarrow & & \downarrow{\scriptstyle p_Y} \\ X/\sim_X & \xrightarrow[{[f]}]{} & Y/\sim_Y \end{array}$$

commutes. What happens if $(\star)$ in the case where $\sim_X$ is the identity relation? (Meaning that $(x_1 \sim_X x_2) \Leftrightarrow (x_1 = x_2)$.)

Exercise 4. Let $(X, \leq)$ be an ordered set. Further, let A and B subsets of X which are bounded above. Show the following statements in the case where the corresponding suprema und infima exist:

(a) $\sup(A \cup B) = \sup(\sup(A), \sup(B))$.

(b) If $A \subset B$, then $\sup(A) \leq \sup(B)$.

(c) If $A \cap B \neq \emptyset$, then $\sup(A \cap B) \leq \inf(\sup(A), \sup(B))$.

Formulate and prove the corresponding statements for subsets C and D of X which are bounded below.

Submission of the exercise sheet: 14.Oct.2019 before the lecture. **Return of the exercise sheet**: 17.Oct.2019 during the exercise classes.