EXERCISES 11: LECTURE FOUNDATIONS OF MATHEMATICS

Exercise 1. An ordered field K is called archimedean if, for all $a, b \in K$ with $a > 0$, there exists $n \in \mathbb{N}$ such that $b < na$. Show that $\mathbb{Q}$ with its natural order is archimedean.

Exercise 2. Show that an ordered field K is archimedean if and only if $\{n \cdot 1 \mid n \in \mathbb{N}\} \subset K$ is not bounded from above.

Exercise 3. Show: For $x \in \mathbb{R}$ with $x > -1$ and $n \in \mathbb{N}$ one has $(1+x)^n \geq 1 + nx$.

Exercise 4. Show that $\mathbb{Q}(\sqrt{2}) = \{a + b\sqrt{2} \mid a, b \in \mathbb{Q}\} \subset \mathbb{R}$ is a field. Here $\sqrt{2} \notin \mathbb{Q}$ denotes a real number with $\sqrt{2}\sqrt{2} = 2$, the addition is

$$(a + b\sqrt{2}) + (c + d\sqrt{2}) = (a + c) + (b + d)\sqrt{2}$$

and the multiplication is

$$(a + b\sqrt{2})(c + d\sqrt{2}) = (ac + 2bd) + (ad + bc)\sqrt{2},$$

for $a, b, c, d \in \mathbb{Q}$.

Submission of the exercise sheet: 10.Dec.2018 before the lecture. **Return of the exercise sheet**: 13.Dec.2018 during the exercise sessions.