Application: The virtual Rasmussen invariant

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Status: Work in progress

Virtual tangles

- Virtual tangle diagrams T_D^k are like virtual link diagrams, but one allows k boundary point on a disk D^2;
- a natural question is if it is possible to extend the virtual Khovanov complex to such diagrams;
- it turns out that another information mod 3 is needed to ensure the complex to be a well-defined chain complex;
- this information is a number $-1, 0, 1$ which depends on different combinatorial conditions;
- as a result of these conditions one gets two extension of the Khovanov complex $[\![\cdot]\!]^*$ and $[\![\cdot]\!]_*$.

Theorem. *The two extensions for a virtual tangle diagram T_D^k are chain homotopic if $k=0$, i.e. for virtual links, or if T_D^k is a classical tangle, i.e. a diagram equivalent to a diagram without virtual crossings.*

The Rasmussen invariant (to be done)

Surprisingly this degenerations gives rise to an very interesting invariant of classical knots, the so-called Rasmussen invariant, an invariant with many nice properties.

In a famous paper J. Rasmussen uses this result and he defines a spectral sequence whose E_2 term is the Khovanov complex and that converges to the Lee complex.

Every term E_i is an invariant of classical knots itself.

This spectral sequence should also exists in the case of virtual knots and therefore an extension of the Rasmussen invariant to virtual knots.

Summary

It is possible to define two different geometric complexes for virtual tangles. Moreover, the classical result about the Lee complex for classical links still holds for virtual links.

This should lead to a virtual Rasmussen invariant with hopefully equivalent nice properties (still to be done).

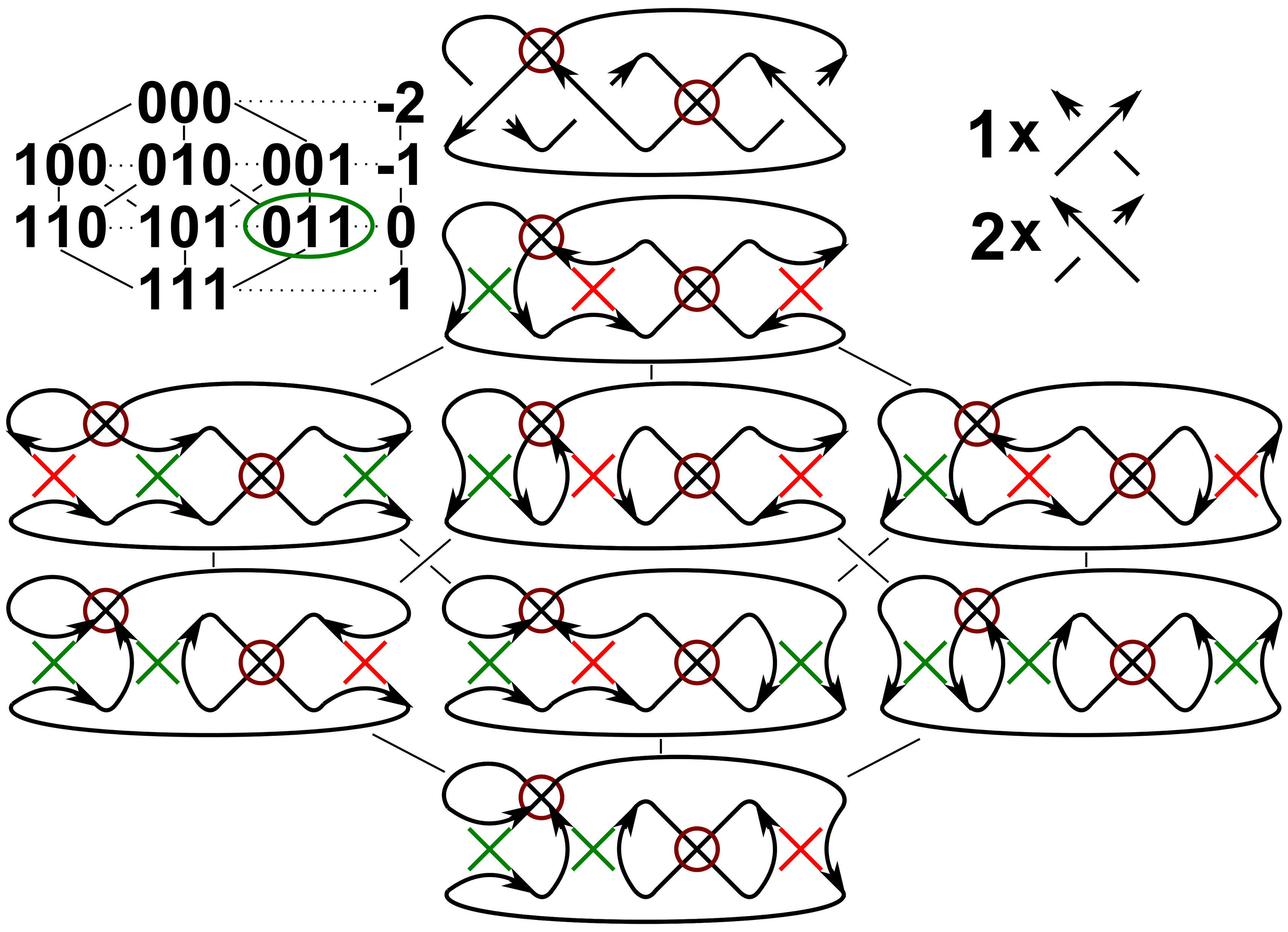
Lee's variant ($h = 0, t = 1$)

As an application of the geometric picture one gets an extension of the Lee complex $[\![L]\!]_{\mathrm{Lee}}$. It known that the classical Lee complex has dimension 2^c for a c-component classical link L. An amazing fact (the virtual complex has zero morphisms over $R = \mathbb{Q}$!) is that this statement is also true for virtual links.

The main idea to proof this is to got to the Karoubi envelope KAR($\mathbf{uCob}^2{}_R(\emptyset)$) of the geometric category and use the extension of the complex to virtual tangle diagrams. The main observation now is that there is a bijection between non-alternating resolutions and generators of the complex.

This results is the main ingredient to proof that the statement is still true for virtual links.

Moreover, if $c = 1$, i.e. in the case of virtual knots, then these non-alternating resolution (and the corresponding generators of the homology) will be in homology degree zero.



For a knot there is only one non-alternating resolution.

Theorem. *A virtual link diagram with c components has, up to orientation, exactly c non-alternating resolutions. The Lee complex of a virtual link diagram is homotopy equivalent to a complex with one generator for each such resolution with zero differentials.*

References

1. Jacob A. Rasmussen, Khovanov homology and the slice genus, *Invent. Math.*, **182** (2010), no. 2, 419–447

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