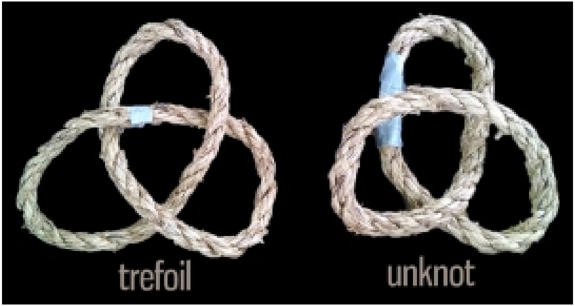
Aspects of (categorical) representation theory

Or: Topology, modular representations and categories

Daniel Tubbenhauer



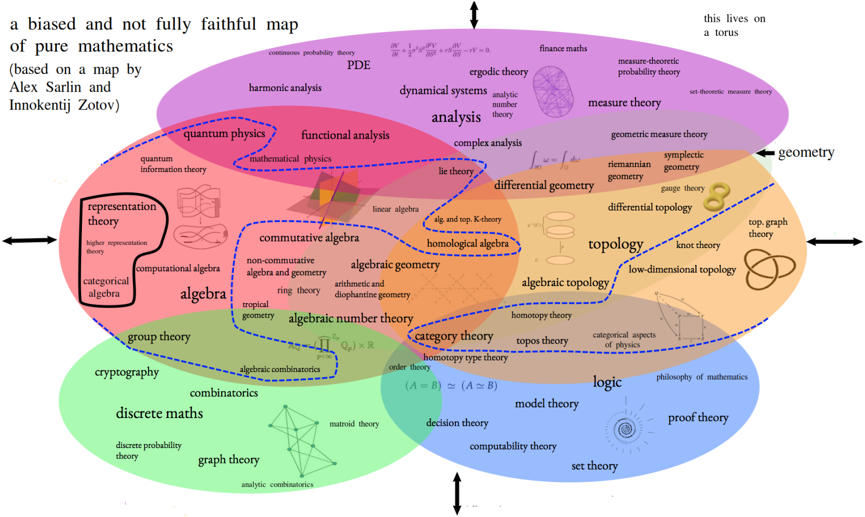
August 2021

Where are we?

a biased and not fully faithful map of pure mathematics

(based on a map by Alex Sarlin and Innokentij Zotov)

this lives on a torus



The six main fields of pure mathematics algebra, analysis, geometry, topology, logic, discrete mathematics

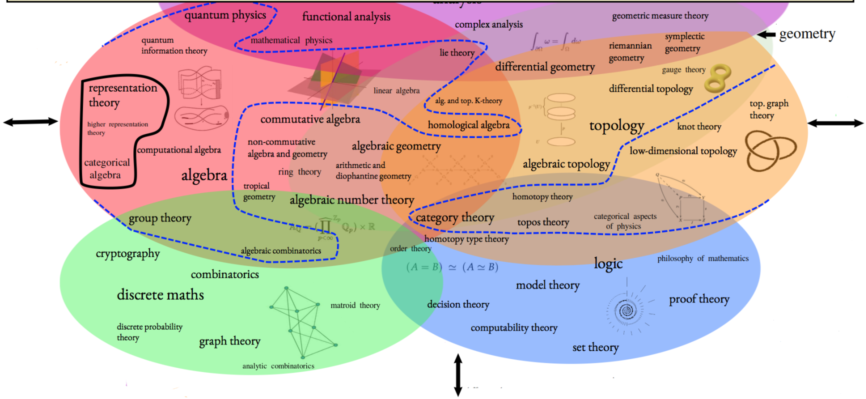
Wh

Today (brief)

Black box. Representation theory and its categorical analog — My research area

Dashed box. Where I like to apply them — My research outreach

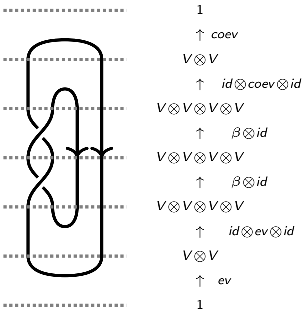
Beyond my current research? Next pdf! — The future (within Aarhus University?)



The six main fields of pure mathematics algebra, analysis, geometry, topology, logic, discrete mathematics

Categorical representation theory and my research

Quantum algebra/topology: representation theory and knots



(a) Put the projection in a Morse position

(b) To each generic horizontal slice associate a representation of a quantum group

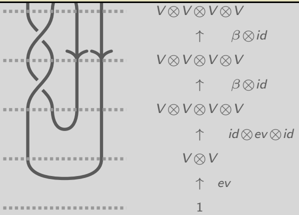
(c) To each basic piece associate a linear map

(d) The whole construction gives a family of invariants

Quantum al

WRT opened a new field of mathematics – quantum algebra

If you ask me at 2am in the morning
"What are you doing for a living?"
I might answer "Quantum algebra"



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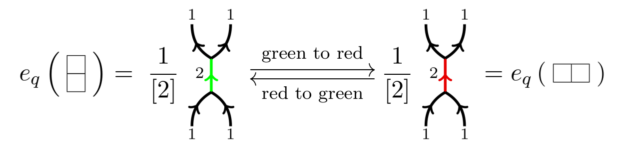
$V\otimes V\otimes V\otimes V$

Quantum algebra is strongly merged with diagrammatic representation theory

For example, there are colorful proofs of symmetries with knot invariants:



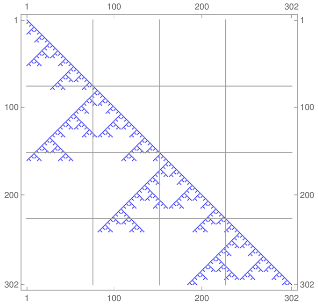
(Picture from one of my papers – using representations of Hecke and super Lie algebras)

(c) To each basic piece associate a linear map

(d) The whole construction gives a family of invariants

Categorical representation theory and my research

Modular representation theory: representation theory and fractals



(a) Take $\mathrm{SL}_2(\overline{\mathbb{F}}_p)$

(b) There are a certain "projective" modules $\mathtt{T}(\lambda)$ for $\lambda \in \mathbb{N}$

(c) There are a certain "standard" modules $\Delta(\lambda)$ for $\lambda \in \mathbb{N}$

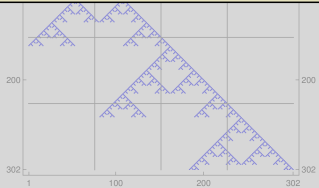
(d) The above is a prototypical example of a "character" $[\mathtt{T}(\lambda) : \Delta(\mu)]$

Categorical representation theory and my research

Modular representation theory: representation theory and fractals

A notoriously hard field of mathematics – modular representation theory

If you ask me at 2am in the morning
"What are you doing for a living?"
I might answer "Modular representation theory"



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Modular representation theory: representation theory and fractals

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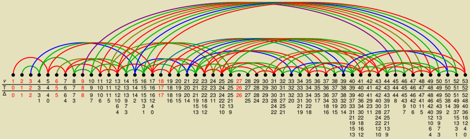
Modular representation theory is strongly merged with quiver representations

For example, quivers underlying modular representation categories look *e.g.*:



(Picture from one of my papers – using representations of Lie groups and Lie algebras)

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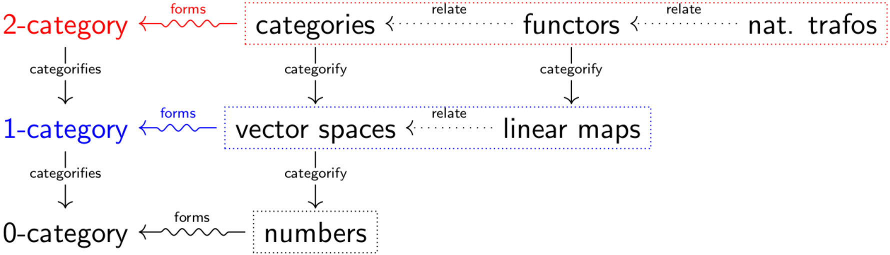
Categorical representation theory and my research

Categorical representation theory: representation theory and category theory



(a) A representation is a functor

(b) A 2-representation ($\approx$ categorical representation) is a 2-functor

(c) Going down one recovers representations and “characters”

(d) Each level contains more information

Categori… …ory

A fairly new field of mathematics – categorical representation theory

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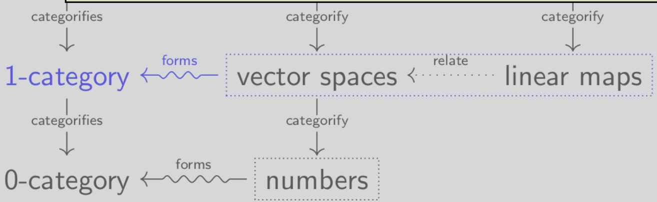
I might answer "Categorical representation theory"



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Categorical representation theory and my research

Categori… …ory

2-cat… …t. trafos

categorifies categorify categorify

A fairly new field of mathematics – categorical representation theory

If you ask me at 2am in the morning
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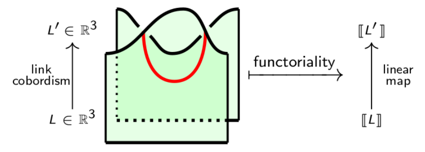
Categorical representation theory is strongly merged with 4-dimensional topology

For example, knot homologies are functorial *e.g.*:



(Picture from one of my papers – using categorical representations to prove functoriality)

(c) Going down one recovers representations and "characters"

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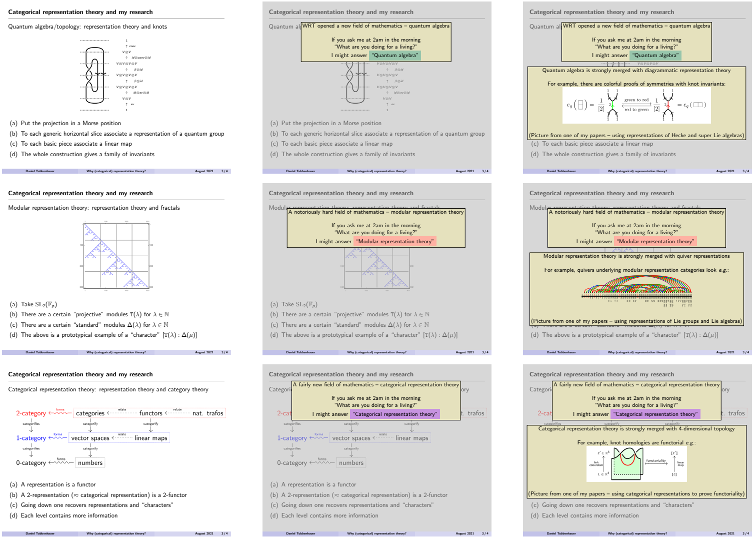
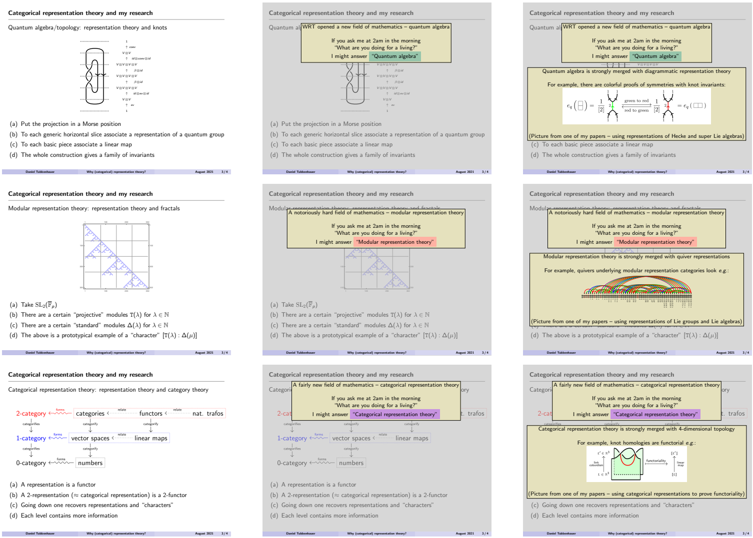


There is still much to do...



Thanks for your attention!