# Pyramids and 2-representations

(Joint with Volodymyr Mazorchuk and Vanessa Miemietz)

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additive category

$\mathcal{A}$ $\rightsquigarrow$ $\mathrm{Com}^-(\mathcal{A})$

the category of right bounded complexes

**strict** monoidal $\dashrightarrow$ monoidal

**losing strictness**

**Q**: How to lift the **strictness** to $\mathrm{Com}^-(\mathcal{A})$?

**A**: **Vague ideas**: try to avoid direct sums

$$\mathbb{Z}\text{-complexes} \xrightarrow{\otimes\,\mathbb{Z}\text{-complexes}} \mathbb{Z}^2\text{-complexes} \xrightarrow{\otimes\,\mathbb{Z}\text{-complexes}} \mathbb{Z}^3\text{-complexes} \xrightarrow{\otimes\ldots} \ldots$$

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$$\begin{tikzcd} \mathbb{Z}\text{-complexes} \arrow[r, dashed, bend right, "\otimes\, \mathbb{Z}\text{-complexes}"'] & \mathbb{Z}^2\text{-complexes} \arrow[r, dashed, bend right, "\otimes\, \mathbb{Z}\text{-complexes}"'] & \mathbb{Z}^3\text{-complexes} \arrow[r, dashed, bend right, "\otimes \dots"'] & \dots \end{tikzcd}$$

$\xRightarrow[\text{the limit}]{\text{taking}}$ "finitary $\mathbb{Z}^\infty$-complexes" $:=$ **Pyramids** $\rightsquigarrow$ a category $\mathcal{P}(\mathcal{A})$

**Want** :

$$\begin{tikzcd}[column sep=huge, row sep=huge]
\mathcal{P}(\mathcal{A}) \arrow[r, dashed, "\mathcal{F}: \text{``taking the total complex''}", "\simeq \text{ biequivalence}"'] \arrow[d, rightsquigarrow] & \mathrm{Com}^-(\mathcal{A}) \arrow[d, rightsquigarrow] & \\
\mathcal{H}(\mathcal{A}) \arrow[r, dashed, "\mathcal{F}", "\simeq"'] & \mathcal{K}^-(\mathcal{A}) & \text{the corr. homo. cat.}
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$(\mathcal{A}, \circ, \mathbb{1})$: an additive **strict** monoidal category with biadditive $\circ$

$\mathcal{C}$: an additive category $\quad \lozenge : \mathcal{A} \times \mathcal{C} \to \mathcal{C}$ a **strict** monoidal action ($SMA$)

$$\begin{tikzcd}
\mathcal{P}(\mathcal{A}) \times \mathcal{P}(\mathcal{C}) \arrow[rr, dashed, "\blacklozenge\text{: a } SMA"] \arrow[d, squiggly] & & \mathcal{P}(\mathcal{C}) \arrow[d, squiggly] \\
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**Application**: Classify all simple transitive 2-reps of $\mathscr{C}_A$, where $A$ is a connected, basic, finite dim'l algebra over $\Bbbk$ (alg. closed).

**Approach**:

$$\begin{tikzcd}
\mathscr{C}_A \arrow[rr, hook] \arrow[dr, "\mathbf{M}"'] & & \mathscr{D}_A \arrow[dl, dashed, "\mathbf{N}"] \\
& \mathscr{A} &
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($\mathscr{A}$: the 2-cat of $\Bbbk$-finitary cats )

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