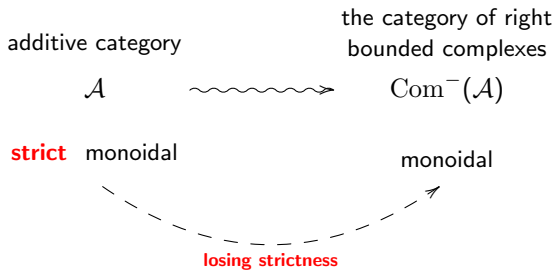


Pyramids and 2-representations

(Joint with Volodymyr Mazorchuk and Vanessa Miemietz)

Xiaoting Zhang
(Uppsala University)

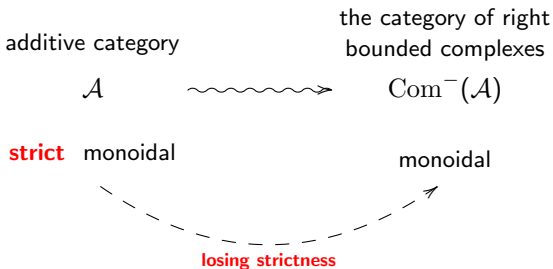
Bonn, November 21, 2017



Q: How to lift the **strictness** to $\text{Com}^-(\mathcal{A})$?

A: **Vague ideas:** try to avoid direct sums

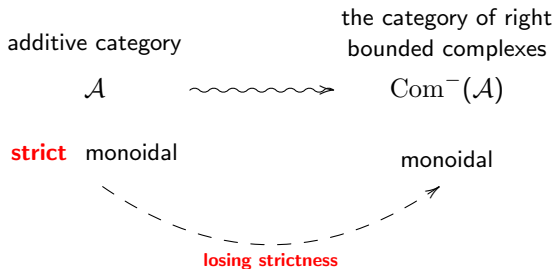




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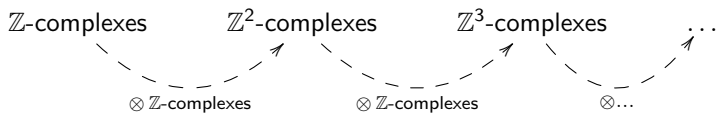
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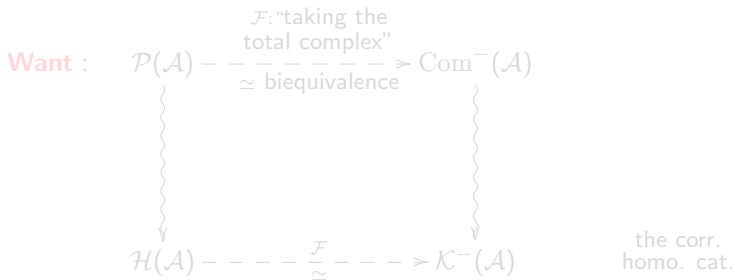


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$\xrightarrow[\text{the limit}]{\text{taking}}$ "finitary \mathbb{Z}^∞ -complexes" := **Pyramids** \rightsquigarrow a category $\mathcal{P}(\mathcal{A})$



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Want :

$$\begin{array}{ccc}
 \mathcal{P}(\mathcal{A}) & \xrightarrow[\simeq]{\mathcal{F}: \text{"taking the total complex"}} & \text{Com}^-(\mathcal{A}) \\
 \downarrow \text{wavy} & & \downarrow \text{wavy} \\
 \mathcal{H}(\mathcal{A}) & \xrightarrow[\simeq]{\mathcal{F}} & \mathcal{K}^-(\mathcal{A})
 \end{array}$$

the corr. homo. cat.

$(\mathcal{A}, \circ, \mathbb{1})$: an additive **strict** monoidal category with biadditive \circ

\mathcal{C} : an additive category $\diamond : \mathcal{A} \times \mathcal{C} \rightarrow \mathcal{C}$ a **strict** monoidal action (**SMA**)

$$\begin{array}{ccc}
 \mathcal{P}(\mathcal{A}) \times \mathcal{P}(\mathcal{C}) & \xrightarrow{\diamond: \text{a SMA}} & \mathcal{P}(\mathcal{C}) \\
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 \mathcal{H}(\mathcal{A}) \times \mathcal{H}(\mathcal{C}) & \xrightarrow{\diamond: \text{a SMA}} & \mathcal{H}(\mathcal{C})
 \end{array}$$

Application: Classify all simple transitive 2-reps of \mathcal{C}_A , where A is a connected, basic, finite dim'l algebra over \mathbb{k} (alg. closed).

Approach:

$$\begin{array}{ccc}
 \mathcal{C}_A & \xrightarrow{\quad} & \mathcal{D}_A \\
 \searrow M & & \swarrow N \\
 & \mathcal{A} &
 \end{array}$$

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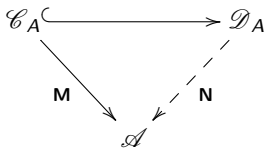
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