Two boundary centralizer algebras for $\mathfrak{gl}(n|m)$

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Introduction

Let A, B be two algebras, W be a $\mathbb{C}$-vector space. We study the centralizing actions

$$A \quad \circlearrowright W \circlearrowleft \quad B$$

	A	W	B
Schur (1905)	$\mathfrak{gl}_n(\mathbb{C})$	$V^{\otimes d}$	sym. gp
Arakawa-Suzuki (1998)	$\mathfrak{sl}_n(\mathbb{C})$	$M \otimes V^{\otimes d}$	degenerate affine Hecke alg.
Daugherty (2010)	$\mathfrak{gl}_n(\mathbb{C})$ or $\mathfrak{sl}_n(\mathbb{C})$	$M \otimes N \otimes V^{\otimes d}$	extend. degenerate Hecke alg
Hill-Kujawa-Sussan (2009)	$\mathfrak{q}_n(\mathbb{C})$	$M \otimes V^{\otimes d}$	affine Hecke-Clifford alg
our case	$\mathfrak{gl}_{n\|m}(\mathbb{C})$	$M \otimes N \otimes V^{\otimes d}$	extend. degenerate Hecke alg

Background

The Lie superalgebra $\mathfrak{g} = \mathfrak{gl}(n|m)$ is the vector space $\mathrm{Mat}_{n+m,n+m}(\mathbb{C})$ with the following $\mathbb{Z}_2$-grading

$$\mathfrak{g}_{\bar{0}} = \left\{ \begin{pmatrix} A & 0 \\ 0 & D \end{pmatrix} | A \in \mathrm{Mat}_{n,n}, D \in \mathrm{Mat}_{m,m} \right\}$$

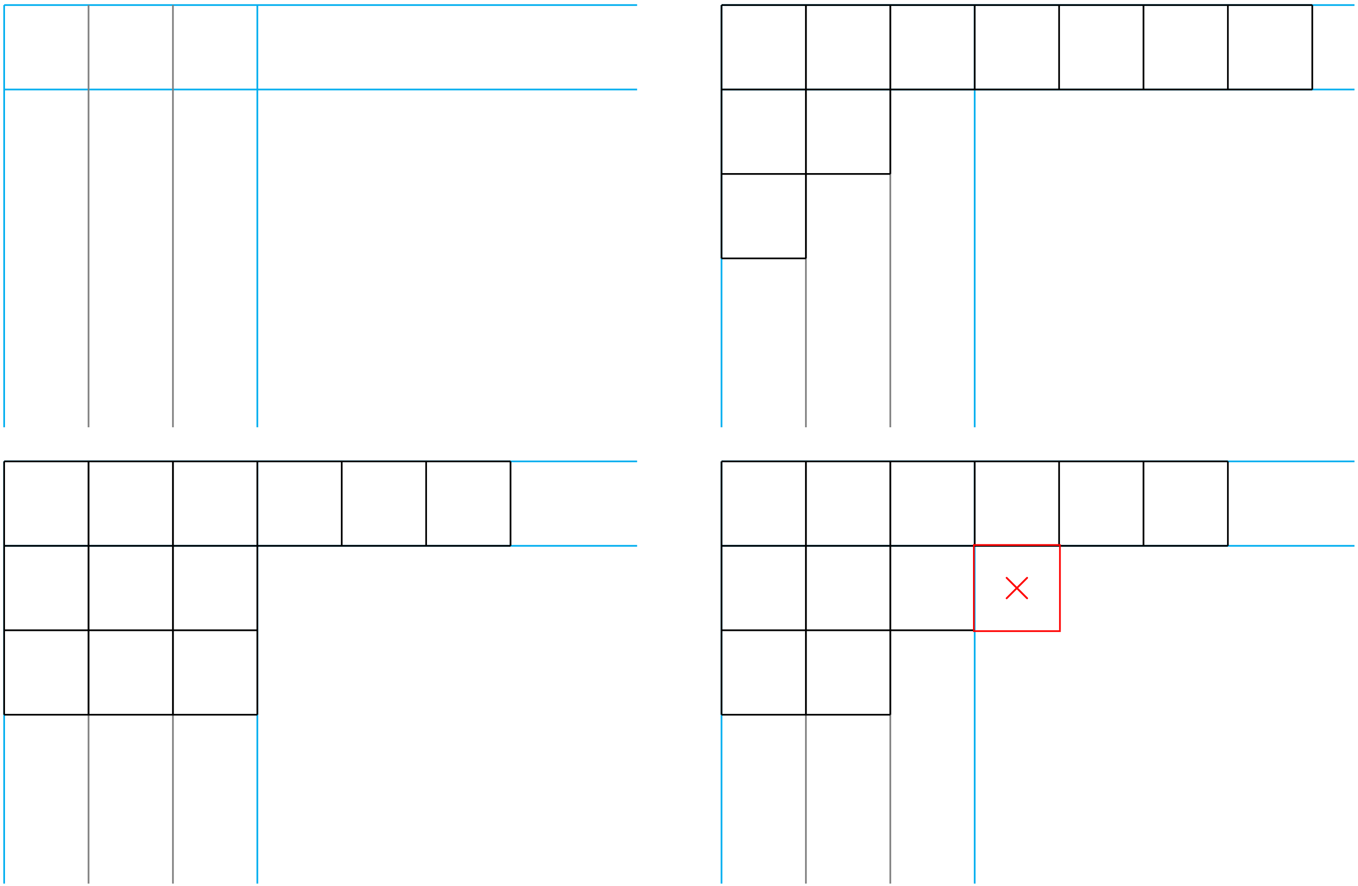
$$\mathfrak{g}_{\bar{1}} = \left\{ \begin{pmatrix} 0 & B \\ C & 0 \end{pmatrix} | B \in \mathrm{Mat}_{n,m}, C \in \mathrm{Mat}_{m,n} \right\}$$

and the Lie brackets

$$[x, y] = xy - (-1)^{i \cdot j} yx, \qquad \forall x \in \mathfrak{g}_{\bar{i}}, y \in \mathfrak{g}_{\bar{j}}$$

$V = \mathbb{C}^{n+m}$: column vectors of height $n+m$
$\mathfrak{g}$ acts by matrix multiplication.
polynomial representations: irreducible summands of $V^{\otimes d}$
(Sergeev, Berele-Regev) are indexed by Young diagrams inside a $(n|m)$-hook.

For $\mathfrak{gl}(1|3)$:



Results

Let the degenrate two-boundary braid group $\mathcal{G}_d$ be generated by

$$\mathbb{C}[x_1, \ldots, x_d], \quad \mathbb{C}[y_1, \ldots, y_d], \quad \mathbb{C}[z_0, \ldots, z_d], \quad \mathbb{C}\Sigma_d$$

under further relations.

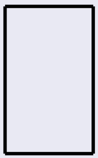
Theorem

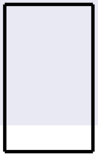
Let M, N be objects in category $\mathcal{O}$ of $\mathfrak{gl}(n|m)$. There is a well-defined action

$$\mathcal{G}_d \to \mathrm{End}_{\mathfrak{gl}(n|m)}(M \otimes N \otimes V^{\otimes d})$$

The (two boundary) extended degenerate Hecke algebra $\mathcal{H}_d^{ext}$ is a quotient of $\mathcal{G}_d$ under further relations.

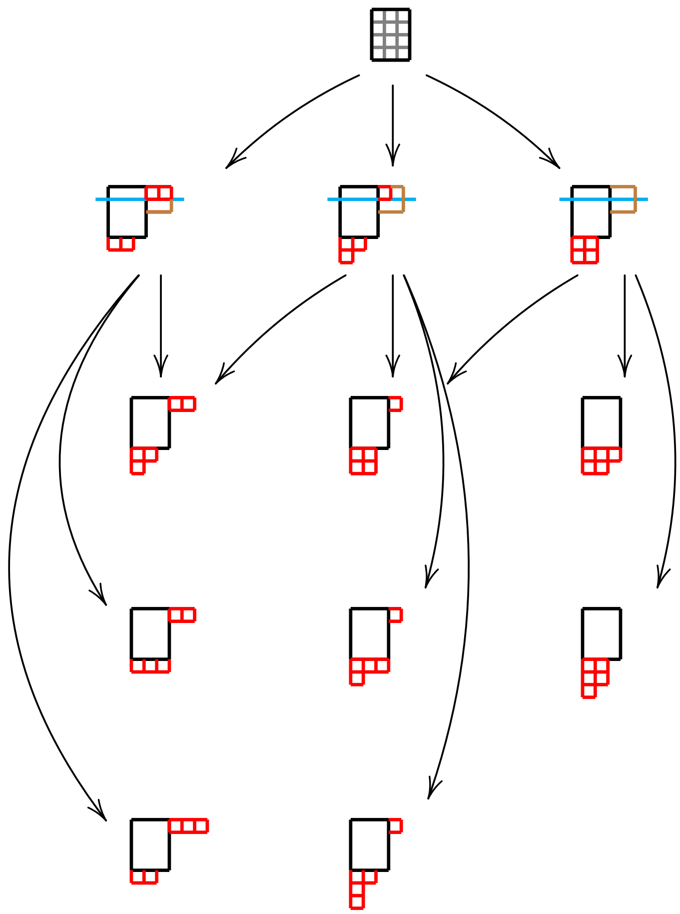
Theorem

Let $L($$)$ *and* $L($$)$ *be two irreducible* $\mathfrak{g}$*-modules labeled by arbituary rectangles inside the* (n, m)*-hook, the above action induces a further action*

$\rho : \mathcal{H}_d^{ext} \to \mathcal{H}_d = \mathrm{End}_{\mathfrak{gl}(n|m)}(L($$) \otimes L($$) \otimes V^{\otimes d})$

$$
\begin{tikzcd}
\mathfrak{gl}(n|m) \curvearrowright & L\left(\boxed{\begin{matrix}\\ \\ \end{matrix}}\right) \otimes L\left(\boxed{}\right) \otimes V^{\otimes d} \arrow[d, phantom, "\cong"] & \curvearrowleft \mathcal{H}_d \\
& \displaystyle\bigoplus_{\substack{\lambda:\text{ hook tableau according}\\ \text{to a combinatorial rule}}} L(\lambda) \otimes \mathcal{L}^{\lambda} & \rho(\mathcal{H}_d^{ext}) \arrow[u, hook]
\end{tikzcd}
$$

where the isomorphism is as $(\mathfrak{gl}(n|m), \mathcal{H}_d)$-bimodules.



Theorem

$\mathcal{L}^\lambda$ admits a basis

$$\{v_T \mid T: \text{semistandard tableaux of the skew shape } \lambda/\mu\}$$

where μ is a diagram inside λ based on certain combinatorial rules.
Furthermore, the polynomial generators z_i act by eigenvalues

$$z_o.v_T = \alpha + \beta|\mathfrak{B}| + \sum_{b\in\mathfrak{B}} 2c(b)$$

$$z_i.v_T = c(i)$$

Where $c(*)$ denotes the content of the box, $\mathfrak{B}$ is a certain set of boxes in λ.

$$\rho : \mathcal{H}_d^{ext} \to \mathcal{H}_d = \mathrm{End}_{\mathfrak{gl}(n|m)}\left(L\left(\boxed{\phantom{\rule{1.2em}{4em}}}\right) \otimes L\left(\boxed{\phantom{\rule{5em}{1.2em}}}\right) \otimes V^{\otimes d}\right)$$

Theorem

$\mathrm{Res}^{\mathcal{H}_d}_{\mathcal{H}_d^{ext}} \mathcal{L}^\lambda$ *is irreducible. Therefore,* $\rho(\mathcal{H}_d^{ext})$ *is a large subalgebra of* $\mathcal{H}_d$*.*

An explicit example..