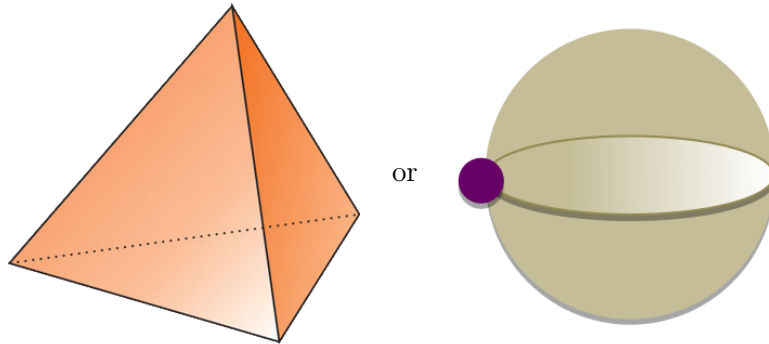


## EXERCISES 7: LECTURE ALGEBRAIC TOPOLOGY

**Exercise 1.** Compute the simplicial and the cellular homology of  $S^2$  using

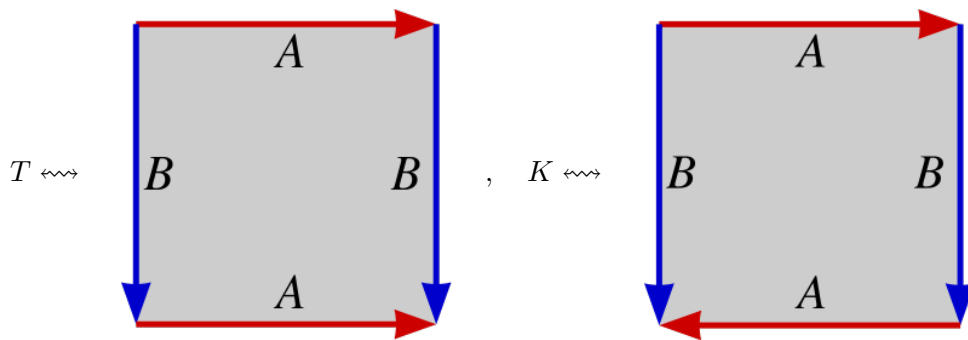


as a simplicial complex, respectively, as a cell complex representing  $S^2$ .

**Exercise 2.** Let  $T$  be the torus and  $K$  be the Klein bottle. Compute  $H_*(T)$  and  $H_*(K)$ .

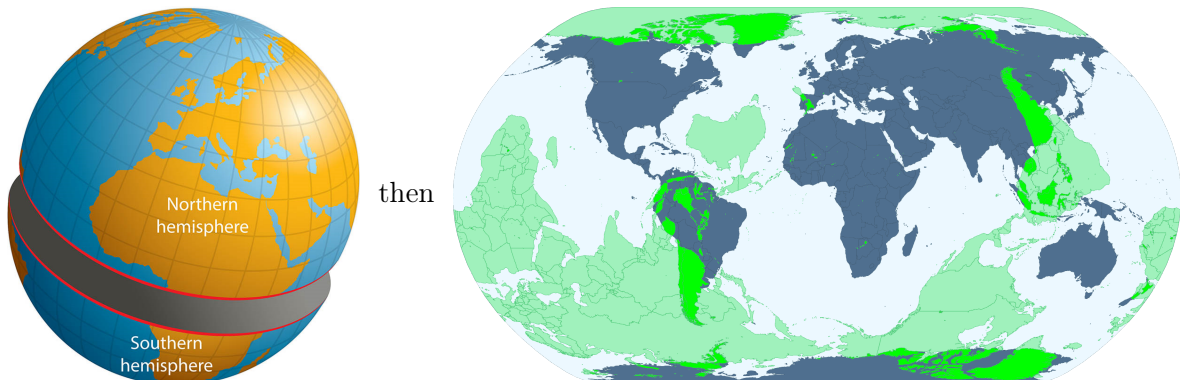
Addendum:

- ▶ Conclude from the exercise that the Euler characteristic  $\chi$  does not distinguish the torus  $T$  from the Klein bottle  $K$ , but the Poincaré polynomial  $P$  does.
- ▶ Hint: To compute homology use the cell structures given by their fundamental polygons

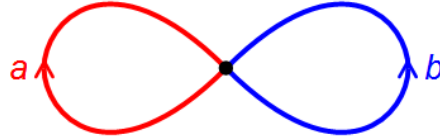


**Exercise 3.** Compute  $H_*(\mathbb{R}P^n)$ .

Hint: Use the inductive cell structure given by antipodal identifications



**Exercise 4.** Let us consider  $S^1 \vee S^1$  using the cell structure

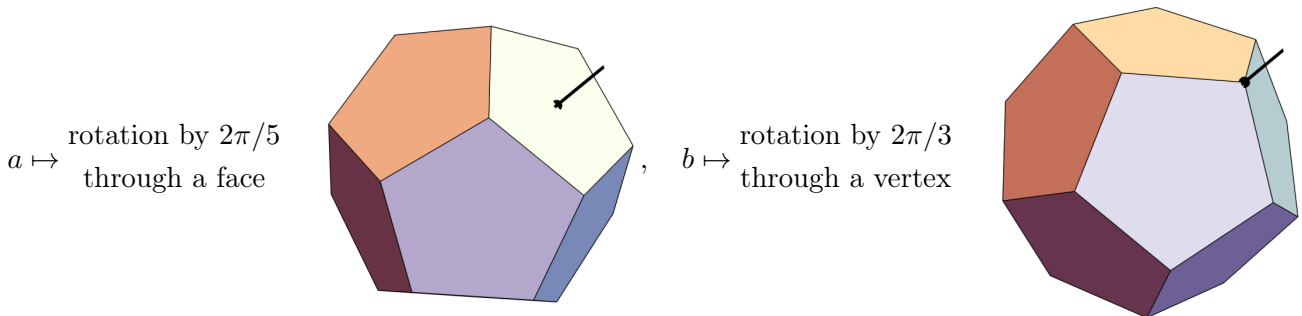


Let  $X$  be the cell complex obtained from  $S^1 \vee S^1$  by attaching two 2-cells via  $a^5b^{-3}$  and  $b^3(ab)^{-2}$ .

1. Show that  $H_1(X, \mathbb{Q})$  is trivial.
2. Show that  $\pi_1(X)$  is not trivial.

Addendum:

- Hint: There is a map from  $\pi_1(X)$  to the symmetry group of the dodecahedron given by



- Hint: The rotational symmetry group of the dodecahedron is  $A_5$

- The exercises are optional and not mandatory. Still, they are highly recommend.
- There will be 12 exercise sheets, all of which have four exercises.
- The sheets can be found on the homepage [www.dtubbenhauer.com/lecture-algtop-2021.html](http://www.dtubbenhauer.com/lecture-algtop-2021.html).
- If not specified otherwise, spaces are topological space, maps are continuous *etc.*
- There might be typos on the exercise sheets, my bad, so be prepared.