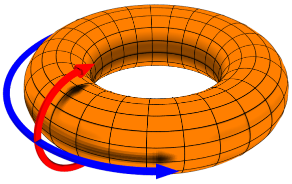
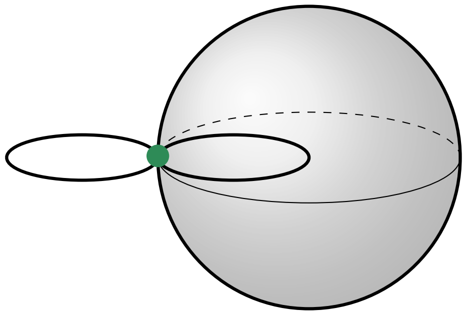
EXERCISES 9: LECTURE ALGEBRAIC TOPOLOGY

Exercise 1. Find $X = \coprod_{i \in I} X_i$ such

$$H^*\left(\coprod_{i \in I} X_i\right) \ncong \bigoplus_{i \in I} H^*(X_i).$$

Hint: Almost everything infinite will do, *e.g.* https://math.stackexchange.com/questions/3943835

Exercise 2. Given the torus $T \cong S^1 \times S^1$ and the space $T' = S^1 \vee S^1 \vee S^2$.

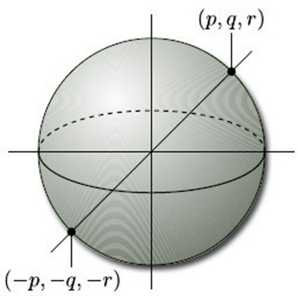
$T \cong$  , $T' =$ 

Compute both, H_* and H^*, for these two spaces.

Exercise 3. Show that $\mathbb{R}P^5$ and $\mathbb{R}P^4 \vee S^5$ have the same (co)homology groups. What about the respective fundamental groups?

Addendum:

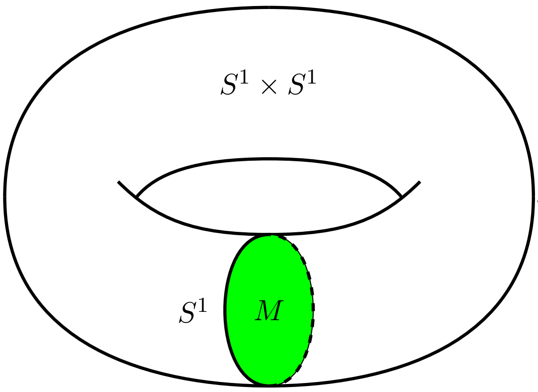
- Hint: For (co)homology it is convenient to use the cell structure given by the antipodal maps:

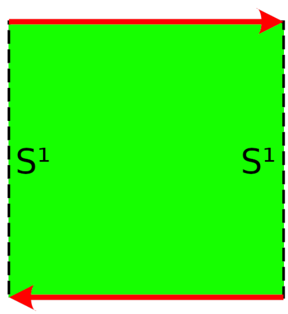


$$\Rightarrow \dots \xrightarrow{0} \mathbb{Z} \xrightarrow{0} \mathbb{Z} \xrightarrow{\cdot 2} \mathbb{Z} \xrightarrow{0} \mathbb{Z} \xrightarrow{\cdot 2} \underline{\mathbb{Z}} \xrightarrow{0} 0.$$

  The antipodal $S^n \to \mathbb{R}P^n$ also helps to compute the fundamental group. Seifert–van Kampen and Mayer–Vietoris (or direct computation) will do the rest.
- Hint: See also https://math.stackexchange.com/questions/3426826

Exercise 4. Compute H_* (or H^*, whatever you prefer) of

$X =$ 

, $M =$ 

where M is a Möbius strip. (Note that the boundary of M is one copy of S^1.)

Addendum: Formally, X is obtained from $S^1 \times S^1$ by gluing a Möbius strip to $S^1 \times \{x_0\}$ via identifying boundaries.

- The exercises are optimal and not mandatory. Still, they are highly recommend.
- There will be 12 exercise sheets, all of which have four exercises.
- The sheets can be found on the homepage www.dtubbenhauer.com/lecture-algtop-2021.html.
- If not specified otherwise, spaces are topological space, maps are continuous *etc.*
- There might be typos on the exercise sheets, my bad, so be prepared.