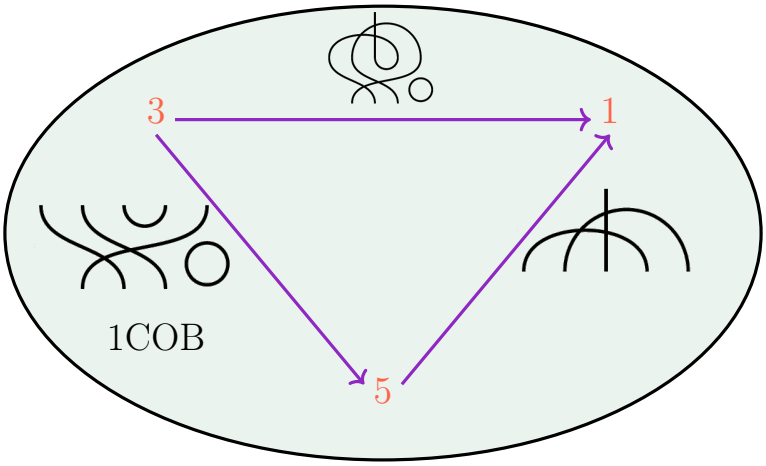
EXERCISES 1: LECTURE CATEGORY THEORY

Exercise 1. Let 1COB be the collection of

- Objects are natural numbers $X \in \mathbb{Z}_{\geq 0}$.
- Arrows are one-dimensional cobordisms.
- Composition $g \circ f$ is stacking g on top of f, *e.g.*



Convince yourself that 1COB is a category.
Addendum:

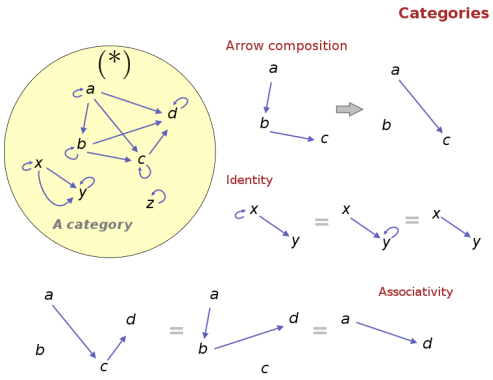
- This means write down a list what one actually would need to check to make the construction of 1COB rigorous.
- How "rigorous" one wants to be is left to the reader ;-)

Exercise 2. Fill in the questions marks in the following table.

Name	Objects	Arrows	Concrete?
SET	Sets	Maps	Yes
1COB	0-manifolds	?	?
nCOB	?	n-manifolds	?
fSET	Finite sets	?	?
pSET	?	Partial maps	?
GROUP	Groups	?	?
TOP	topological spaces	?	?
oTOP	?	continuous open maps	?
$\mathbb{K}$VECT	?	$\mathbb{K}$-linear map	?
$\mathbb{K}$MAT	?	$\mathbb{K}$-valued matrices	?
$\bullet$	?	?	?
$(*)$	?	?	?

Addendum:

- The category (*) is

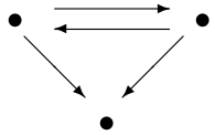


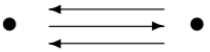
  Note that this is the only example where identities are illustrated (they are the loops at the vertices).
- In this exercise "concrete" means based on maps, meaning that arrows have the underlying structure of maps in SET.

Exercise 3. Determine which of the following graphs could be interpreted as categories.

(1) 

(2) 

(3) 

(4) 

Addendum:
- Recall how to read these graphs: vertices correspond to objects and edges to arrows.
- Recall also that identity arrows are not illustrated. In particular, graphs without loops still might give rise to categories.

Exercise 4. Decide which of the categories listen in Exercise 2 are small, locally small or large.
Addendum:
- A category is small if objects and arrows are sets (and not proper classes).
- A category is locally small if arrows form sets.
- A category is large if objects or arrows are proper classes.

- The exercises are optimal and not mandatory. Still, they are highly recommend.
- There will be 12 exercise sheets, all of which have four exercises.
- The sheets can be found on the homepage www.dtubbenhauer.com/lecture-ct-2022.html.
- The distinction between "large classes" and "small classes (sets)" turns out is crucial for many categorical considerations, but somehow makes the language more cumbersome. If not stated otherwise (which happens rarely and will be easy to spot), then all set-theoretical issues will be strategically ignored in the lecture and on the exercise sheets.
- There might be typos on the exercise sheets, my bad, so be prepared.