EXERCISES 10: LECTURE CATEGORY THEORY

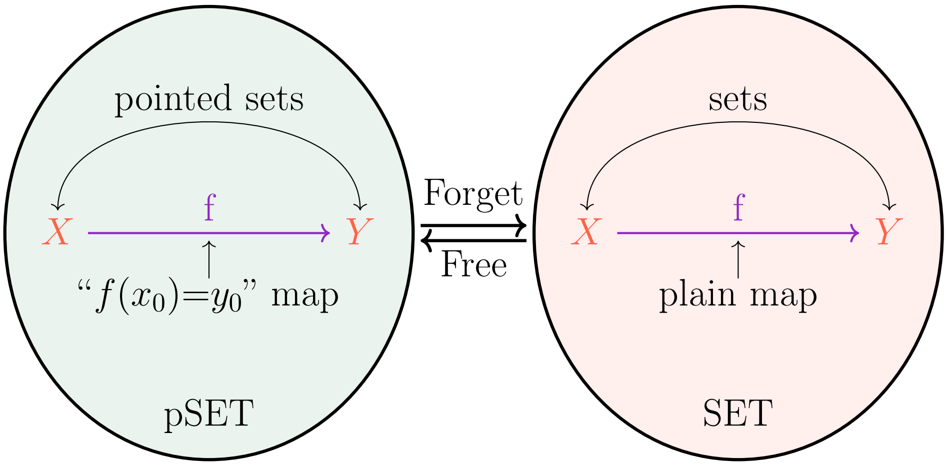
Exercise 1. Here is a joke (see also https://stackoverflow.com/questions/3870088):

A monad is a monoid in the category of endofunctors, what's the problem?'

Justify the joke. Mathematically, of course ;-)

Hint: What is meant is to prove that a monad is a monoid in the category of endofunctors. It is also recommended to use search the internet.

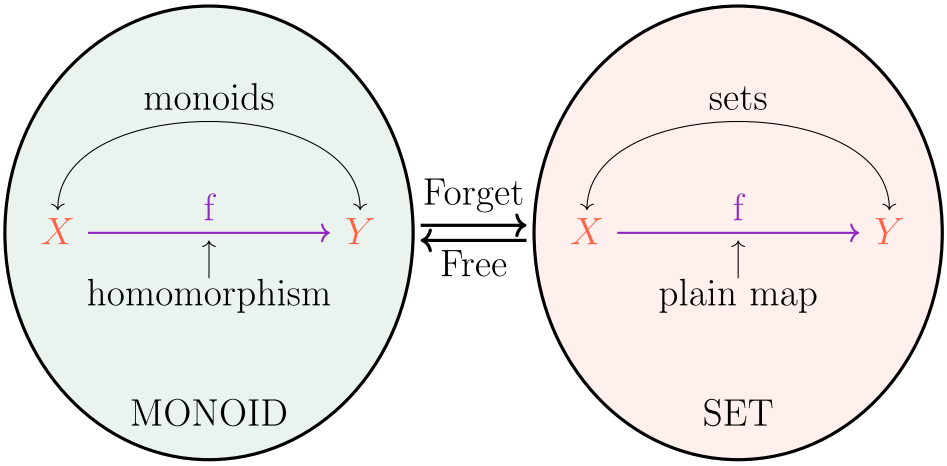
Exercise 2. Let pSET be the category of pointed sets, *i.e.* pairs (X, x_0) of a set X and an element $x_0 \in X$ with arrows being maps satisfying $f(x_0) = y_0$. There is a $(Free, Forget)$ adjunction



Describe the associated monad, sometimes called the maybe monad.

Hint: Free(X)=$(X \sqcup \{x_0\}, x_0)$, that is, the free functor adjoints a new, formal point x_0.

Exercise 3. Describe the free monoid monad, *i.e.* the monad arising from the $(Free, Forget)$ adjunction



Are there other monads in the same spirit?

Exercise 4. Let $Sym\colon \mathbb{Z}\mathrm{MOD} \to \mathbb{Z}\mathrm{MOD}$ be the symmetric algebra functor, sending M to $Sym(M) = \bigoplus Sym^k(M)$, where $Sym^k(M)$ is the kth symmetric power.

Show that Sym is a monad on $\mathbb{Z}\mathrm{MOD}$ and describe its algebras.

Hint: See also https://en.wikipedia.org/wiki/Monad_(category_theory) under the subtitle "Algebras over the symmetric monad".

- The exercises are optimal and not mandatory. Still, they are highly recommend.
- There will be 12 exercise sheets, all of which have four exercises.
- The sheets can be found on the homepage www.dtubbenhauer.com/lecture-ct-2022.html.
- The distinction between "large classes" and "small classes (sets)" turns out is crucial for many categorical considerations, but somehow makes the language more cumbersome. If not stated otherwise (which happens rarely and will be easy to spot), then all set-theoretical issues will be strategically ignored in the lecture and on the exercise sheets.
- There might be typos on the exercise sheets, my bad, so be prepared.