LECTURE: CATEGORY THEORY

Disclaimer

Nobody is perfect, and I might have written or said something silly. If there is any doubt, then please check the references or contact me. All questions welcome!

What?
Consider the following question:

Why category theory? It is abstract nonsense and completely useless, right?

Well, there are many views and reasons to study category theory, and here is a short and biased list:

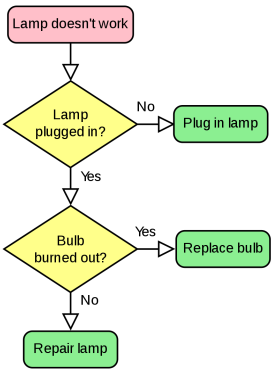
- **Organization of concepts.**
  The book [ML98] start with the sentence "Category theory starts with the observation that many properties of mathematical systems can be unified and simplified by a presentation with diagrams of arrows.". Similarly, quoting [AL91]: "In addition to its direct relevance to theoretical knowledge and current applications, category theory is often used as an (implicit) mathematical jargon rather than for its explicit notions and results. [...] In other words, many different formalisms and structures may be proposed for what is essentially the same concept; the categorical language and approach may simplify through abstraction, display the generality of concepts, and help to formulate uniform definitions."
  In other words, organizes many concepts in one language. It is like the bird's-eye view: by ignoring details, the bird's-eye view reveals hidden symmetries:

  This description is based on a very true quote in [Le14]: "Category theory takes a bird's eye view of mathematics. From high in the sky, details become invisible, but we can spot patterns that were impossible to detect from ground level.".

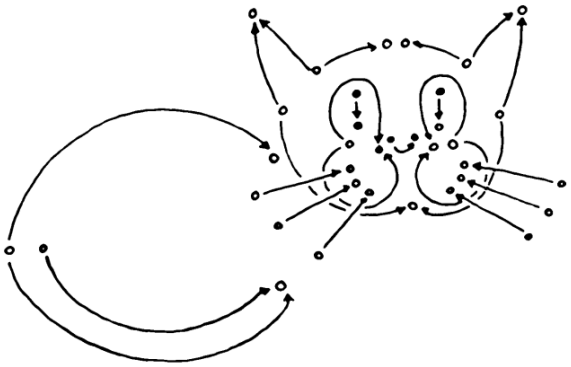
- **Organization of ideas.**
  Category theory is not just abstract nonsense, but can be applied in various parts of mathematics, the sciences and more generally. This aspect of category theory is not highlighted as often as it deserves to be. But times change and [FS19] starts with: "Category theory is becoming a central hub for all of pure mathematics. It is unmatched in its ability to organize and layer abstractions, to find commonalities between structures of all sorts, and to facilitate communication between different mathematical communities. But it has also been branching out into science, informatics, and industry. We believe that it has the potential to be a major cohesive force in the world, building rigorous bridges between disparate worlds, both theoretical and practical.".
  Nowadays category theory has entered for example computer science (think about Haskell), philosophy and logic (at least it is proposed, see e.g. [La17]), physics (see e.g. [BL11]) and

many more. Not so much as a really practical tool, but more like a flowchart:

Flowcharts



Categories



a) Flowcharts A symbolism that allows one to organize complicated facts

b) Category theory A symbolism that allows one to organize complicated facts

The flowchart description above is copied from Wikipedia, but it fits also perfectly well for category theory. In other words, category theory is not just a way to organize concepts, but also to organize ideas in general.

- **Beauty.**
  The book [AHS90] starts with a poem, describing category theory and many people's views on it spot on:

  There's a tiresome young man in Bay Shore.
  When his fiancée cried, 'I adore
  The beautiful sea',
  He replied, 'I agree,
  It's pretty, but what is it for?'

  Morris Bishop

  Nothing to add!

Who?

Fourth semester students in Mathematics interested in a mixture of (linear) algebra and discrete mathematics, but everyone is welcome.

Where and when?

- Time and date for the lecture.
  - Every Monday from 12:00–14:00.
  - Online.
  - First lecture: Monday 21.Feb.2022. Last lecture: Monday 09.May.2022.
- Time and date for the tutorials.
  - Every Friday from 12:00–14:00.
  - Online.
  - First tutorial: Friday 25.Feb.2022. Last tutorial: Friday 13.May.2022.

Material for the lecture

- The lecture is a mix of various sources. The main source is [ML98], and the lecture follows the list of topics presented therein. However, the lecture takes a different perspective compared to [ML98] and potentially reading either of [AHS90], [AL91], [FS19], [Le14], [Mi14], [Ri16] or [Si11] should be beneficial.
- Website www.dtubbenhauer.com/lecture-ct-2022.html

- Prerecorded lectures on the "What is...category theory?" playlist here: www.youtube.com/c/VisualMath/playlists
- Exercise sheets are available on the course website.

Preliminary Schedule.

- The beginnings – What is...category theory? (21.Feb.2022)
- Diagrams in categories – Commuting and alike. (28.Feb.2022)
- Functors I – The basics about functors. (07.Mar.2022)
- Functors II – Natural transformations and equivalence. (14.Mar.2022)
- Yoneda – Yoneda lemma and Yoneda embedding. (21.Mar.2022)
- Limits I – Examples of limits. (28.Mar.2022)
- Limits II – Universal properties and limits abstractly. (04.Apr.2022)
- Adjoint functors I – The algebraic approach. (11.Apr.2022)
- Adjoint functors II – The diagrammatic approach. (18.Apr.2022)
- Monoids I – Monads and their modules. (25.Apr.2022)
- Monoids II – Monoidal categories. (02.May.2022)
- Whats next? – Some outlook including diagrammatics. (09.May.2022)

References

[AL91] A. Asperti, G. Longo. Categories, types, and structures. An introduction to category theory for the working computer scientist. Foundations of Computing Series. MIT Press, Cambridge, MA, 1991.

[AHS90] J. Adámek, H. Herrlich, G.E. Strecker. Abstract and concrete categories: the joy of cats. Reprint of the 1990 original [Wiley, New York; MR1051419]. Repr. Theory Appl. Categ. No. 17 (2006), 1–507. URL: http://katmat.math.uni-bremen.de/acc/acc.pdf

[BL11] J.C. Baez, A.D. Lauda. A prehistory of n-categorical physics. Deep beauty, 13–128. Cambridge Univ. Press, Cambridge, 2011. URL: https://arxiv.org/abs/0908.2469

[FS19] B. Fong, D.I. Spivak. An invitation to applied category theory. Seven sketches in compositionality. Cambridge University Press, Cambridge, 2019. xii+338 pp. URL: https://arxiv.org/abs/1803.05316

[La17] E. Landry et al. Categories for the working philosopher. Edited by Elaine Landry. Oxford University Press, Oxford, 2017. xiv+471 pp.

[Le14] T. Leinster. Basic category theory. Cambridge Studies in Advanced Mathematics, 143. Cambridge University Press, Cambridge, 2014. viii+183 pp. URL: https://arxiv.org/abs/1612.09375

[ML98] S. Mac Lane. Categories for the working mathematician. Second edition. Graduate Texts in Mathematics, 5. Springer-Verlag, New York, 1998. xii+314 pp.

[Mi14] B. Milewski. Category Theory for Programmers. Collected from the series of blog posts. URL: https://bartoszmilewski.com/2014/10/28/category-theory-for-programmers-the-preface/ URL: https://github.com/hmemcpy/milewski-ctfp-pdf

[Ri16] E. Riehl. Category Theory in Context. Dover Publications, 2016. URL: https://math.jhu.edu/~eriehl/context/

[Si11] H. Simmons. An introduction to category theory. Cambridge University Press, Cambridge, 2011. x+226 pp.

Daniel Tubbenhauer, The University of Sydney, School of Mathematics and Statistics, F07 - Carslaw Building, Office Carslaw 827, NSW 2006, Australia, www.dtubbenhauer.com

Email address: daniel.tubbenhauer@sydney.edu.au