Potential master or Ph.D. project of I WANT YOU in 2022

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Key information

Candidate. I WANT YOU.

Email. YOUR EMAIL.

Research areas. Algebra, representation theory, and categorical algebra, more specifically tensor categories and their semisimplifications.

Title. *"Semisimplifications of tensor categories"*.

First read. [EGNO15] (diagonally) and [TV17] (diagonally).

The notion of a monoidal category dates back to Mac Lane who developed this categorical analog of a monoid ("a group without inverses") in the 1960s of the last century.

In the same vein, tensor categories should be thought of as counterparts of rings in the world of categories. They are ubiquitous in noncommutative algebra and representation theory, and also play an important role in many other areas of mathematics, such as algebraic geometry, algebraic topology, number theory, the theory of operator algebras, mathematical physics, and theoretical computer science.

In fact, the theory of tensor generalizes the theory of Hopf algebras – a notion which first appeared in topology as an algebraic notion capturing the structure of cohomology rings. Much later it was realized that Hopf algebras could be viewed as algebraic structures arising from tensor categories with a fiber functor, through the so-called reconstruction theory. Since then, the theory of Hopf algebras has increasingly been becoming a part of the theory of tensor categories, and in proving some of the more recent results on Hopf algebras tensor categories play a fundamental role.

Nowadays many problems in algebra, geometry, topology and physics lead to tensor categories—categories with a product similar to the tensor product of vector spaces. Some simple examples are: Vector spaces, of course. Group graded vector spaces. The category of finite-dimensional representations of a group. The category of finite-dimensional representations of a Hopf algebra. Algebraic representations of an algebraic group. Continuous representations of a topological group.

However, the most interesting tensor categories are non-semisimple. This is where the paper [EO18] enters the game: It introduces a certain procedure, called semisimplification, that has tensor categories as an input and semisimple tensor categories as an output.

Minimal goal. Summarize the paper [EO18] in your own words.
Average goal. Add some examples of semisimplifications which are not mentioned or discussed in [EO18].
Optimal goal (for Ph.D.). Address some of the open questions mentioned below.
Key. Be concise with the basics. Be precise with the basics on tensor categories; in particular, the paper [EO18] is very densely written and the arguments therein need to be explained carefully.

The thesis in details – minimal goal

The minimal master project should be structured as follows.

- Prepare all the background, *e.g.* [EGNO15, Chapters 1, 2 and 4], carefully.
- Summarize [EO18]. Hereby, be careful that the paper is a bit sloppy when it comes to details.
- Understand and summarize the final examples [EO18, Section 5 onward].

The thesis in details – average goal

As above, but add:

- Study semisimplifications of other examples, *e.g.* of modules categories of the Taft Hopf algebra, see [CVOZ14].
- What are semisimplifications of other Deligne categories, see *e.g.*[CW12]? Can you nail them down?
- Add more details and examples regarding the appendix in [EO18]. For this have *e.g.*a look at [Ost03] (and check via google scholar who cites this paper).

The thesis in details – optimal goal

Here are some open question which (if time suffices) deserve further study.

- What are semisimplifications of Soergel categories, see [Lib17]. Again, an you nail them down?
- Can semisimplifications of dihedral Soergel categories [Eli16], [MT19] be related (via *e.g.* [Eli17]) to low-dimensional topology?
- Can you relate the constructions in [EO18] to [TW21], *i.e.* can you see the semisimplification directly on the quiver from [TW21]?

References

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