MAT595: SEMINAR REPRESENTATION THEORY OF ALGEBRAS – PLAN

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Disclaimer

Please do not hesitate to contact me in case of questions. (The contact data can be found below.) Never forget to give many examples during your talk!

What?

The philosophy is: Many interesting questions in mathematics can be studied by looking at their linearization *a.k.a.* their linear shadows. This is the main idea behind representation theory. For example, one can study algebras (which are non-linear) and their linear shadows, which is the main objective of the seminar.

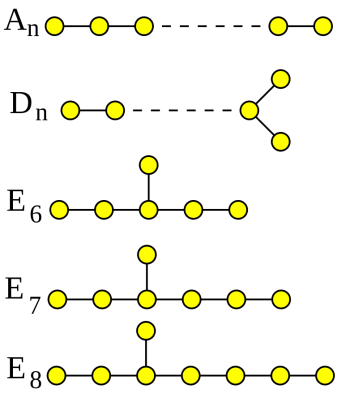
The seminar follows the book [Sc14].

Who?

BSC or MSC or PhD students in Mathematics interested in a mixture of linear algebra and combinatorics, but everyone is welcome.

Where and when?

- Time and date.
  - Every Monday from 13:00–14:45.
  - Room Y21F70, University Zurich, Institute of Mathematics.
  - First meeting: Monday 02.Mar.2020. Last meeting: Monday 18.May.2020. Preliminary meeting: Friday 14.Feb.2020.
- Exceptions.
  - The talk from Monday the 02.Mar.2020 was canceled and will take place on 09.Mar.2020 at the usual time 13:00-14:45.
  - The talk from Monday the 09.Mar.2020 is moved to 09.Mar.2020, 15:00-16:45, room Y27H25.
- Website http://www.dtubbenhauer.com/seminar-algebras-2020.html



Schedule and some details.

▷ 1th talk "Representations of quivers I".

- **Speaker.** Mariya.
- **Date.** 09.Mar.2020, 13:00–14:45.
- **Topic.** The basic concepts regarding quivers.
- **Plan.** Introduce the notions of quivers [Sc14, Definition 1.1] and their representations [Sc14, Definition 1.2]. Explain the two corresponding examples. Move on to morphisms of representations, and present [Sc14, Proposition 1.1], doing the examples carefully. Then direct sums of representations, with [Sc14, Theorem 1.2] being the main goal of the talk. On your way, explain [Sc14, Categories 1 and 2].
- **Main goals.** Present and explain the Krull–Schmidt theorem [Sc14, Theorem 1.2] carefully, while focusing presenting the examples.
- **Note.** There are plenty of examples in the book, and examples are important. Skip proofs if necessary. The "Goal of representation theory" should be presented.
- **Literature.** [Sc14, Sections 1.1 and 1.2], [Sc14, Problem 1.2].

▷ 2th talk "Representations of quivers II".

- **Speaker.** Therese.
- **Date.** 09.Mar.2020, 15:00–16:45.
- **Topic.** The first instance of an Auslander–Reiten quiver.
- **Plan.** Explain all ingredients needed to understand short exact sequences and their properties, *cf.* [Sc14, Section 1.3], in particular, [Sc14, Proposition 1.8] whose proof you should summarize, not giving all the details. Do the same with [Sc14, Section 1.4], do not miss the twin theorems [Sc14, Theorem 1.10 and 1.13]. Spent 20 minutes to finally explain [Sc14, Section 1.5], in particular, [Sc14, Example 1.14 and Problem 1.3.1]. Do as many examples as possible – the problem section should be another source for such examples.
- **Main goals.** The main point is to present [Sc14, Section 1.5].

- **Note.** You have quite a bit of material; skip proofs if necessary.
- **Literature.** [Sc14, Sections 1.3, 1.4 and 1.5], as well as [Sc14, Problem 1.3.1].

▷ 3th talk " Projective and injective representations I".

- **Speaker.** Caroline and Rebekka
- **Date.** 16.Mar.2020, 13:00–14:45.
- **Topic.** Introduce the concepts of simple, projective and injective representations.
- **Plan.** Follow [Sc14, Section 2.1]: First, introduce simple, projective and injective representations, and present [Sc14, Examples 2.3 and 2.4]. Follow the book with [Sc14, Proposition 2.9] being the next main result. Finally, present [Sc14, Corollary 2.14] as a consequence of [Sc14, Theorem 2.11]. On your way do [Sc14, Problem 2.2].
- **Main goals.** The representability of representation via projectives [Sc14, Theorem 2.11].
- **Note.** Please do as many examples as possible, you have plenty of time.
- **Literature.** [Sc14, Section 2.1] and [Sc14, Problem 2.2].

▷ 4th talk "Projective and injective representations II".

- **Speaker.** Angela
- **Date.** 23.Mar.2020, 13:00–14:45.
- **Topic.** Projective and injective resolutions.
- **Plan.** Follow [Sc14, Section 2.2]: First, introduce projective and injective resolutions, which always exists [Sc14, Theorem 2.15]. Second, present [Sc14, Examples 2.5 and 2.6]. Then define the projective cover and the injective envelope, which are unique [Sc14, Proposition 2.18]. Among the easiest representations are free ones, which comes next, and [Sc14, Corollary 2.21] is crucial. Finish with the radical, and present [Sc14, Problem 2.3.1].
- **Main goals.** Explain the main properties of projective and injective representations and why they are so much easier than others.
- **Note.** The presented concepts are not easy to digest, so please do not skip examples.
- **Literature.** [Sc14, Section 2.2] and [Sc14, Problem 2.3.1].

▷ 5th talk "Projective and injective representations III".

- **Speaker.** Genta
- **Date.** 30.Mar.2020, 13:00–14:45.
- **Topic.** Duality and extensions.
- **Plan.** Follow [Sc14, Sections 2.3 and 2.4]. More precisely, introduce the duality of quivers which show how projectives and injectives are related [Sc14, Proposition 2.29], but also do [Sc14, Examples 2.7, 2.8 and 2.9]. After some more properties

of the Nakayama functors, move on to extensions with the definition and [Sc14, Examples 2.7, 2.8 and 2.9]

- **Main goals.** Carefully explain in what sense projectives and injectives are the same.
- **Note.** You have quite a bit of "categorical material" to cover which needs to be explained carefully. If you are running short on time, then rather skip some proofs.
- **Literature.** [Sc14, Sections 2.3 and 2.4].

▷ 6^{th} talk "Auslander–Reiten quivers".

- **Speaker.** Luca and Marius
- **Date.** 06.Apr.2020, 13:00–14:45.
- **Topic.** Auslander–Reiten quivers of type A.
- **Plan.** Start with the quiver of type A, and explain the knitting algorithm and how to compute τ-orbits. Illustrate both in type A. Then explain the second and third methods, using Coxeter matrices and, very cute, some geometrical constructions [Sc14, Section 3.1.3]. Finish with [Sc14, Section 3.1.4], *i.e.* explain how to compute various numerical data such as Ext dimensions.
- **Main goals.** Explain the ideas of Auslander–Reiten quivers via examples.
- **Note.** Examples are key for your talk.
- **Literature.** The whole chapter [Sc14, Section 3.1].

▷ 7^{th} talk "Gabriel's theorem".

- **Speaker.** Mario and Vivien
- **Date.** 27.Apr.2020, 13:00–14:45.
- **Topic.** More about ADE.
- **Plan.** State Gabriel's theorem [Sc14, Theorem 3.1], but omit the proof for the time being. Present the Auslander–Reiten quiver of type D [Sc14, Section 3.2] and do [Sc14, Problem 3.1]. Finish in the last 30 minutes by sketching the proof of Gabriel's theorem [Sc14], the main ingredient being [Sc14, Theorem 8.6]. To this end, explain what one needs to know to understand this theorem, *i.e.* quadratic forms on quivers [Sc14, Section 8.2].
- **Main goals.** Gabriel's theorem [Sc14, Theorem 3.1].
- **Note.** You have plenty of time, so do the examples carefully. You can also present some more examples from the problem section.
- **Literature.** [Sc14, Sections 3.2 and 3.3] and also [Sc14, Problem 3.1]. Moreover, bits of [Sc14, Chapter 8].

▷ 8^{th} talk "Algebras and modules I".

- **Speaker.** Emre and Lily

- **Date.** 04.May.2020, 13:00–14:45.
- **Topic.** The path algebra of a quiver.
- **Plan.** First, recall some basic notions from ring theory [Sc14, Section 4.1]. (Be brief.) Go on to algebras [Sc14, Section 4.2] and explain all examples in [Sc14, Example 4.2]. Then comes then main concept, the path algebra, where you should spend some time explaining it, *e.g.* via [Sc14, Example 4.3]. After [Sc14, Proposition 4.4], finish by explaining modules of algebras with [Sc14, Example 4.6] being the last thing to be discussed.
- **Main goals.** Define algebras and modules and make the connection to quivers.
- **Note.** At one point while you discuss algebras present [Sc14, Problem 4.5].
- **Literature.** [Sc14, Chapter 4] to (including) [Sc14, Example 4.6], as well as [Sc14, Problem 4.5].

▷ 9^{th} talk "Algebras and modules II".

- **Speaker.** Stephan
- **Date.** 11.May.2020, 13:00–14:45.
- **Topic.** Idempotents.
- **Plan.** Start with [Sc14, Example 4.7] and explain Nakayama's lemma and its consequences. Up next is the decomposition of modules using idempotents [Sc14, Section 4.4], which is crucial. Continue and explain all ingredients needed to prove the important [Sc14, Corollary 4.20].
- **Main goals.** Explain and state [Sc14, Corollary 4.20].
- **Note.** Please do not forget to do the example, *e.g.* [Sc14, Examples 4.12 and 4.13].
- **Literature.** [Sc14, Example 4.7] to the end of the chapter.

▷ 10^{th} talk "Quivers with relations".

- **Speaker.** Lucy and Simon
- **Date.** 18.May.2020, 13:00–14:45.
- **Topic.** Answering the question "Why do we care?".
- **Plan.** Summarize [Sc14, Sections 5.1 and 5.2]. Also mention the remaining sections in this chapter, but be brief. Rather present some problems as [Sc14, Problems 5.1, 5.2 and 5.3] and [Sc14, Problems 5.7, 5.8 and 5.9].
- **Main goals.** Explain [Sc14, Remark 5.3] and also [Sc14, Theorem 5.4].
- **Note.** This is a bit open-ended. Summarize the chapter [Sc14, Chapter 5] without going to much into details.
- **Literature.** [Sc14, Chapter 5].

References

[Sc14] R. Schiffler. *Quiver representations.* CMS Books in Mathematics/Ouvrages de Mathématiques de la SMC. Springer, Cham, 2014.

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