MAT534: SEMINAR REPRESENTATION THEORY OF FINITE GROUPS – PLAN

DANIEL TUBBENHAUER

Disclaimer

Please do not hesitate to contact me in case of questions. (The contact data can be found below.) Never forget to give many examples during your talk!

What?

The philosophy is: Groups are symmetries, and symmetries are interesting, but hard. So we want to translate groups into (way easier) algebra. Preferable into linear algebra. This is the main idea behind representation theory, i.e. replace (usually non-linear) groups by their linear shadow (matrices).

The seminar follows the book [St12].

Who?

Bachelor students interested in a mixture of linear algebra and group theory, but everyone is welcome.

Where and when?

- Time and date.
  - Every Monday from 13:00–14:45.
  - Room Y27H28, University Zurich, Institute of Mathematics.
  - First meeting: Monday 23.Sep.2019. Last meeting: Monday 25.Nov.2019.
- Preliminary meeting: Monday 16.Sep.2019, 13:00–14:45, room Y27H28.
- Website http://www.dtubbenhauer.com/seminar-fgroups-2019.html

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$\ydiagram{3}$	1	1	1
$\ydiagram{2,1}$	2	0	-1
$\ydiagram{1,1,1}$	1	-1	1

Schedule and some details.

▷ 1^{th} talk "Group representations I".
  - **Speaker.** Mariya.

- **Date.** 23.Sep.2019, 13:00–14:45.
- **Topic.** The basic notions from representation theory.
- **Plan.** Introduce representations, equivalence of these, subrepresentations, irreducible=simple representations, direct sums etc., cf. [St12, Section 3.1]. Then present the illustrative [St12, Proposition 3.1.19]. Finish by explaining completely reducible representations.
- **Main goals.** Present and explain [St12, Table 3.1] carefully.
- **Note.** There are plenty of examples in the book, and examples are important. Skip proofs if necessary. [St12, Exercise 3.4] is great.
- **Literature.** [St12, Chapter 2] (for the basics), [St12, Section 3.1].

▷ 2^{th} talk "Group representations II".

- **Speaker.** Samuel.
- **Date.** 30.Sep.2019, 13:00–14:45.
- **Topic.** Maschke's theorem.
- **Plan.** The first main goal is to prove [St12, Proposition 3.2.3] and, even more important [St12, Proposition 3.2.4] (the proof of the latter is long – streamline it). These have the consequence [St12, Theorem 3.2.8], which need to be explained carefully. In particular, why its an analog of the spectral theorem.
- **Main goals.** Explain unitarizability and its consequences.
- **Note.** Unitarizability is an important concept and needs to be explained carefully.
- **Literature.** [St12, Section 3.2].

▷ 3^{th} talk "Morphisms of representations".

- **Speaker.** Stephan.
- **Date.** 07.Oct.2019, 13:00–14:45.
- **Topic.** Intertwiners a.k.a. morphisms of representations.
- **Plan.** Introduce the concept of morphisms of representations (this also includes [St12, Exercise 4.1]) and then proof Schur's lemma [St12, Lemma 4.1.6] carefully. The explain [St12, Corollary 4.1.8] and deduce some applications to linear algebra [St12, Corollaries 4.1.9 and 4.1.10].
- **Main goals.** Explain the classification of irreducible representations of abelian groups [St12, Corollary 4.1.8 and Exercise 3.4] and its proof.
- **Note.** You have plenty of time – so go slowly. Schur's lemma [St12, Lemma 4.1.6] is very important and can not be stressed often enough.
- **Literature.** [St12, Section 4.1].

▷ 4^{th} talk "Characters I".

- **Speaker.** Adrian.
- **Date.** 14.Oct.2019, 13:00–14:45.

- **Topic.** Character theory of finite groups – orthogonality.
- **Plan.** After introducing the group algebra, prove [St12, Proposition 4.2.2] carefully. Move on to a variant of Schur's lemma [St12, Proposition 4.2.3] and use it to show the main statement [St12, Theorem 4.2.8]. Conclude by deducing some consequences e.g. [St12, Proposition 4.2.10].
- **Main goals.** Proof orthogonality [St12, Theorem 4.2.8].
- **Note.** The "averaging trick" [St12, Proposition 4.2.2] is also very important. Discuss what happens in positive characteristic.
- **Literature.** [St12, Section 4.2].

▷ 5th talk "Characters II".

- **Speaker.** Mario.
- **Date.** 21.Oct.2019, 13:00–14:45.
- **Topic.** Character theory of finite groups – the main players.
- **Plan.** Discuss the notions of class functions and characters, and why the character is an invariant of representations [St12, Proposition 4.3.4]. Move on an discuss the canonical basis of the class functions [St12, Proposition 4.3.8] and its consequence e.g. [St12, Corollary 4.3.10] or [St12, Theorem 4.3.14 and Corollary 4.3.15]. Conclude by discussing the case of S_3.
- **Main goals.** The amazing [St12, Corollaries 4.3.10 and 4.3.15].
- **Note.** The case of S_3 is important and you should keep in mind that you need some time to do it carefully.
- **Literature.** [St12, Section 4.3].

▷ 6th talk "The regular representation".

- **Speaker.** Vivien.
- **Date.** 28.Oct.2019, 13:00–14:45.
- **Topic.** The group action on itself a.k.a the regular representations.
- **Plan.** Introduce the regular representation and deduce its character theory [St12, Proposition 4.4.3]. Explain the main point – that the regular representation contains all irreducibles [St12, Theorem 4.4.4]. From it one obtains a basis of class functions [St12, Theorem 4.4.7], and, more importantly, [St12, Corollary 4.4.8]. Finish by explaining the character table and [St12, Section 3.5].
- **Main goals.** The numerical condition in [St12, Corollary 4.4.5] is great and needs to be explained.
- **Note.** You have a lot of material. So you need to be brief at some points. Focus on the main statements, and skip proofs if necessary.
- **Literature.** [St12, Sections 3.4 and 3.5].

▷ 7th talk "A few applications I".

- **Speaker.** Genta.
- **Date.** 04.Nov.2019, 13:00–14:45.
- **Topic.** A version of Lagrange's theorem for representations.
- **Plan.** The first part is a reminder (or introduction) to algebraic integers [St12, Section 6.1], which can be kept brief. Then the first important statement is that the characters are algebraic [St12, Corollary 6.2.1] which leads to [St12, Theorem 6.2.4]. Finish with by hinting the first applications, i.e. mention [St12, Corollary 6.2.6] as a primer for next time.
- **Main goals.** Prove [St12, Theorem 6.2.4].
- **Note.** Take your time to prove [St12, Theorem 6.2.4], it is important and the analog of Lagrange's theorem.
- **Literature.** [St12, Sections 6.1 and 6.2; to Corollary 6.2.6 (excluded)].

▷ 8^{th} talk "A few applications II".

- **Speaker.** Genta.
- **Date.** 11.Nov.2019, 13:00–14:45.
- **Topic.** A proof of Burnside's theorem.
- **Plan.** This talk is about giving some applications of the theory which we developed so far. This includes [St12, Corollaries 6.2.6 and 6.2.8], but also the main player [St12, Theorem 6.3.11]. (Note that it took about 60 years to give a "representation theory free" proof of Burnside's theorem – stress this since it illustrates how useful representation theory can be.)
- **Main goals.** Prove and explain a few main applications of representation theory to finite groups, i.e. [St12, Corollaries 6.2.6 and 6.2.8, Theorem 6.3.11].
- **Note.** Focus on the main ideas and stress that it took 60 years to find a purely group-theoretical proof of Burnside's theorem.
- **Literature.** [St12, Sections 6.2 and 6.3, from Corollary 6.2.6 onward].

▷ 9^{th} talk "A few applications III".

- **Speaker.** Genta.
- **Date.** 18.Nov.2019, 13:00–14:45.
- **Topic.** How representation theory covers group actions.
- **Plan.** Explain (or recall) the concept of a group action on a set and related notions as transitivity. Important is also the notion of fixed points, see [St12, Above Proposition 7.1.11]. Explain how this can be linearized [St12, Definition 7.2.1], and explain some nice properties of these permutation representations given by the various propositions in [St12, Section 7.2]. Conclude with [St12, Theorem 7.2.11].
- **Main goals.** Show how the concept of group representations can be used to deduce theorems about group actions.
- **Note.** Please find time to cover [St12, Example 7.2.13].

- **Literature.** [St12, Sections 7.1 and 7.2].

▷ 10$^{\text{th}}$ talk "The symmetric group".

- **Speaker.** Mariya.
- **Date.** 25.Nov.2019, 13:00–14:45.
- **Topic.** Representations of symmetric groups.
- **Plan.** This talk is rather open. Present the main ideas how one can reduce the representation theory of the symmetric group to "pure combinatorics". The keywords are Young diagrams, tableaux tabloids etc.
- **Main goals.** Summarize [St12, Chapter 10].
- **Note.** Do not give proofs or too many details, but be rather very explicit and give examples.
- **Literature.** [St12, Chapter 10].

References

[St12] B. Steinberg. *Representation theory of finite groups. An introductory approach.* Universitext. Springer, New York, 2012. xiv+157 pp.

D.T.: Institut für Mathematik, Universität Zürich, Winterthurerstrasse 190, Campus Irchel, Office Y27J32, CH-8057 Zürich, Switzerland, www.dtubbenhauer.com

Email address: daniel.tubbenhauer@math.uzh.ch