LECTURE: QUANTUM INVARIANTS OF LINKS

What?

Quantum invariants are more than just topological invariants needed to tell objects apart. They build bridges between topology, algebra, number theory and quantum physics helping to transfer ideas, and stimulating mutual development. They also have a deep and interesting connection to representation theory, in particular, to representations of quantum groups.

In this course we will introduce these objects from different perspectives: skein and representation theoretic. We will start with the Jones polynomial, study its properties, and then move to the categorification of this polynomial discovered by Khovanov. In the second part (i.e. this lecture) of the class we will explain its connections to representation theory following the ideas of e.g. Reshetikhin–Turaev, and then explain how the categorification also arises from very natural constructions in categorical representation theory.

The lecture follows several texts, e.g. [BS11], [EGNO], [HV19] or [TV17].

Who?

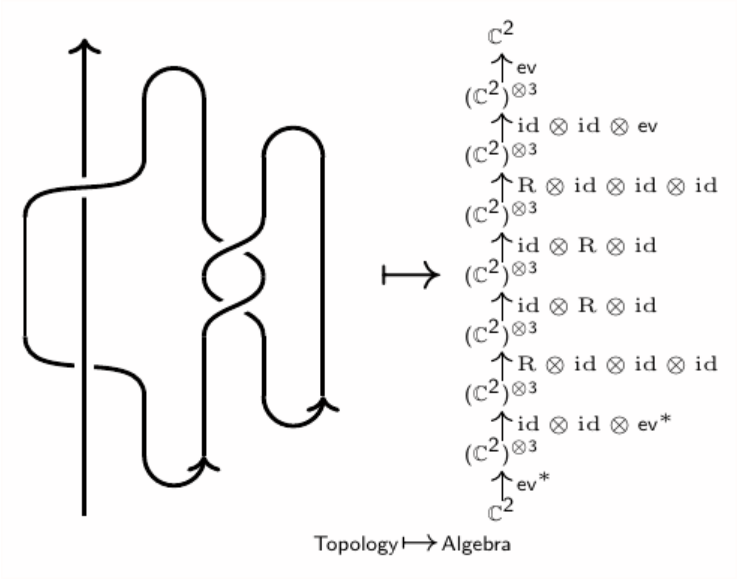
MSC or PhD students in Mathematics interested in a mixture of (linear) algebra, topology and category theory, but everyone is welcome.

Where and when?

- ▶ Time and date.
  - Every Monday from 10:15–12:00.
  - Online, see https://www.youtube.com/channel/UC7wuTzExlC6RDhCjmNdd4Hw.
  - First lecture: Monday 01.Feb.2021. Last lecture: Monday 05.Apr.2021.
- ▶ Website http://www.dtubbenhauer.com/lecture-qinv-2021.html

Preliminary Schedule.

- ▷ Categories—definitions, examples and graphical calculus. (01.Feb.2021)
- ▷ Monoidal categories I – definitions, examples and graphical calculus. (08.Feb.2021)
- ▷ Monoidal categories II – more graphical calculus. (15.Feb.2021)
- ▷ Pivotal categories – definitions, examples and graphical calculus. (22.Feb.2021)
- ▷ Braided categories – definitions, examples and graphical calculus. (01.Mar.2021)
- ▷ Additive, linear and abelian categories – definitions and examples. (08.Mar.2021)
- ▷ Fiat and tensor categories – enrich the concepts from before. (15.Mar.2021)
- ▷ Fiat, tensor and fusion categories – definitions and classifications. (22.Mar.2021)
- ▷ Fusion and modular categories – definitions and graphical calculus. (29.Mar.2021)
- ▷ Quantum invariants – a diagrammatic approach. (05.Apr.2021)



References

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[HV19] C. Heunen, J. Vicary. Categories for Quantum Theory: An Introduction. Oxford Graduate Texts in Mathematics, **28**. Oxford University Press, Oxford, 2019. xii+336 pp. Comes close: http://www.cs.ox.ac.uk/people/jamie.vicary/IntroductionToCategoricalQuantumMechanics.pdf

[TV17] V. Turaev, A. Virelizier. Monoidal categories and topological field theory. Progress in Mathematics, **322**. Birkhäuser/Springer, Cham, 2017.

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