**EXERCISES 6: LECTURE REPRESENTATION THEORY**

**Exercise 1.** Here is the character table of $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/3\mathbb{Z}$:

```
----------------------------------
Class |   1  2    3    4    5    6
Size  |   1  1    1    1    1    1
Order |   1  2    3    3    6    6
----------------------------------
p  =  2   1  1    4    3    3    4
p  =  3   1  2    1    1    2    2
----------------------------------
X.1   +   1  1    1    1    1    1
X.2   +   1 -1    1    1   -1   -1
X.3   0   1  1    J -1-J -1-J    J
X.4   0   1 -1 -1-J    J   -J  1+J
X.5   0   1 -1    J -1-J  1+J   -J
X.6   0   1  1 -1-J    J    J -1-J
```

How does the character table of an arbitrary finite abelian group looks like?

**Exercise 2.** Let $\chi$ be a nontrivial simple character. Show that $\sum_{g\in G} \chi(g) = 0$.
Verify this in the $S_4$ example:

```
-----------------------
Class |   1  2  3  4  5
Size  |   1  3  6  8  6
Order |   1  2  2  3  4
-----------------------
p  =  2   1  1  1  4  2
p  =  3   1  2  3  1  5
-----------------------
X.1   +   1  1  1  1  1
X.2   +   1  1 -1  1 -1
X.3   +   2  2  0 -1  0
X.4   +   3 -1 -1  0  1
X.5   +   3 -1  1  0 -1
```

**Exercise 3.** Let $f\colon G \to H$ be a surjective group homomorphism and let $\phi\colon H \to \mathrm{GL}_n(\mathbb{C})$ be a simple $H$-representation. Prove that $\phi \circ f$ is a simple $G$-representation.

**Exercise 4.** Show that if $g \in G$ is nontrivial, then there is a simple representation $\phi$ with $\phi(g) \neq \mathrm{id}$ (id is the identity matrix).

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- The exercises are optimal and not mandatory. Still, they are highly recommend.
- There will be 12 exercise sheets, all of which have four exercises.
- The sheets can be found on the homepage www.dtubbenhauer.com/lecture-rt-2022.html.
- Slogan: "Everything that could be finite is finite, unless stated otherwise.". For example, groups are finite and representations are on finite dimensional vector spaces.
- There might be typos on the exercise sheets, my bad, so be prepared.