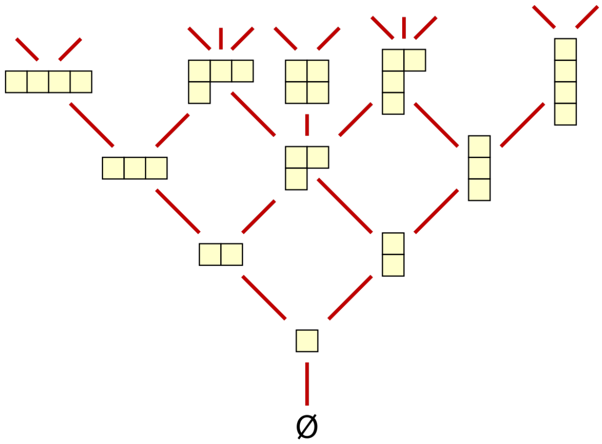
EXERCISES 9: LECTURE REPRESENTATION THEORY

Exercise 1. The picture



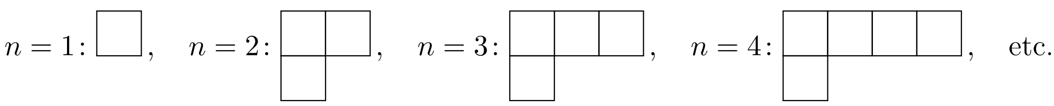
shows the so-called Young lattice. Explain its importance for the representation theory of the symmetric group.

Exercise 2. Here is the character table of S_5:

```
-----------------------------
Class |   1  2  3  4  5  6  7
Size  |   1 10 15 20 30 24 20
Order |   1  2  2  3  4  5  6
-----------------------------
p  =  2   1  1  1  4  3  6  4
p  =  3   1  2  3  1  5  6  2
p  =  5   1  2  3  4  5  1  7
-----------------------------
X.1   +   1  1  1  1  1  1  1
X.2   +   1 -1  1  1 -1  1 -1
X.3   +   4 -2  0  1  0 -1  1
X.4   +   4  2  0  1  0 -1 -1
X.5   +   5  1  1 -1 -1  0  1
X.6   +   5 -1  1 -1  1  0 -1
X.7   +   6  0 -2  0  0  1  0
```

Verify that this is indeed the character table of S_5 using the theory of Specht modules.

Exercise 3. The following Young diagrams are of the form $(n-1,1)$:



Show that the associated Specht module is the quotient of the permutation representation of S_n on $\mathbb{C}^n$ by the fixed vector $(1,\dots,1)$.

Exercise 4. Here is the character table of S_3:

```
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Class |   1  2  3
Size  |   1  3  2
Order |   1  2  3
----------------
p  =  2   1  1  3
p  =  3   1  2  1
----------------
X.1   +   1  1  1
X.2   +   1 -1  1
X.3   +   2  0 -1
```

What happens to the Specht module for the representation of dimension two, so the partition $(2,1)$, in the case where the underlying field is of characteristic three?

- The exercises are optimal and not mandatory. Still, they are highly recommend.
- There will be 12 exercise sheets, all of which have four exercises.
- The sheets can be found on the homepage www.dtubbenhauer.com/lecture-rt-2022.html.
- Slogan: "Everything that could be finite is finite, unless stated otherwise.". For example, groups are finite and representations are on finite dimensional vector spaces.
- There might be typos on the exercise sheets, my bad, so be prepared.