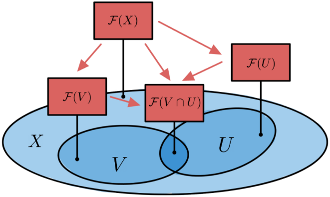
What are...sheaves, take 2?

Or: A discrete version

Recall from the previous video



Sheaves, in a limited way for now and only on some (complex) manifold M:

(i) A presheaf $\mathcal{F}$ is a collection

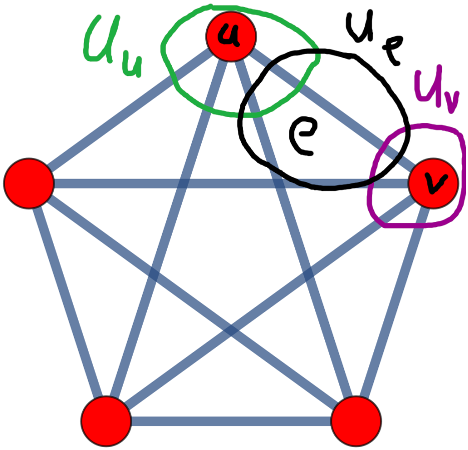
$$\mathcal{F}(U) = \{f\colon U \to \mathbb{C} \text{ satisfying condition X (e.g. holomorphic)}\}$$

for any nonempty open subset $U \subset M$ such that the restriction $f|_{U'} \in \mathcal{F}(U')$ for all $f \in \mathcal{F}(U)$ and $U' \subset U$

(ii) A presheaf $\mathcal{F}$ is a sheaf if for any open set $U \subset M$ and any open cover U_i: $f \in \mathcal{F}(U)$ if $f|_{U_i} \in \mathcal{F}(U_i)$

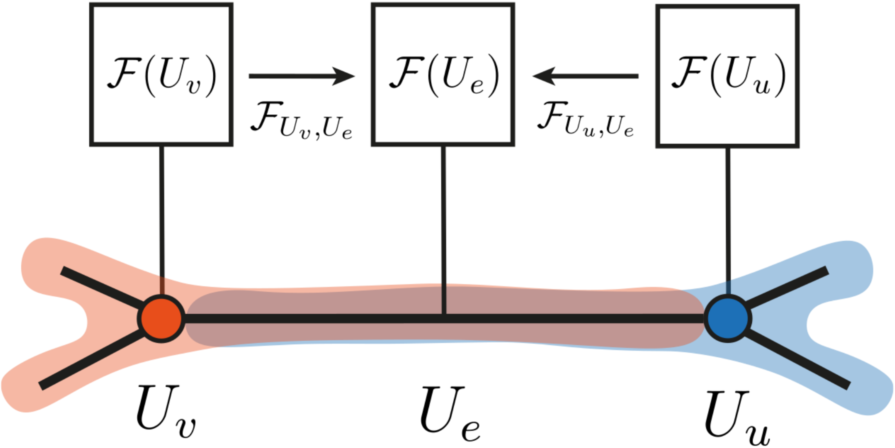
- Staring at the above definition, what catches the eye ?
- Sheaf = a way to associate data to a space, e.g. ($\mathcal{F}(U)$ to U) to M
- Today The same idea applied in a discrete context

Open sets on graphs



- Graph/Network = a collection of vertices and edges
- Open sets for vertices and edges as above
- So what is a sheaf on a graph?

Presheaves on graphs



- Pick a finite dimensional vector space $\mathcal{F}(v)$ for vertex
- Pick a finite dimensional vector space $\mathcal{F}(e)$ for edge
- Pick restriction maps $\mathcal{F}(u) \to \mathcal{F}(e)$ and $\mathcal{F}(v) \to \mathcal{F}(e)$ for $e: u \to v$

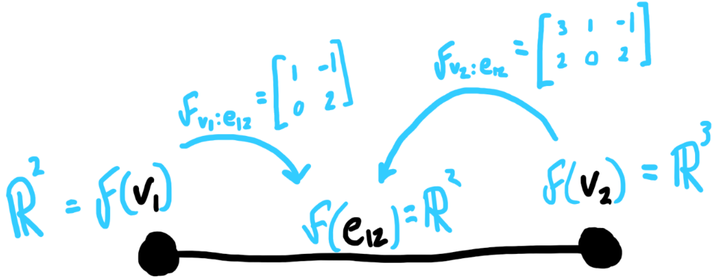
For completeness: A formal statement

$\mathcal{F}$ is a **sheaf** (of finite dimensional vector spaces) on a graph G if:

(i) $\mathcal{F}$ is a presheaf as on the previous slide

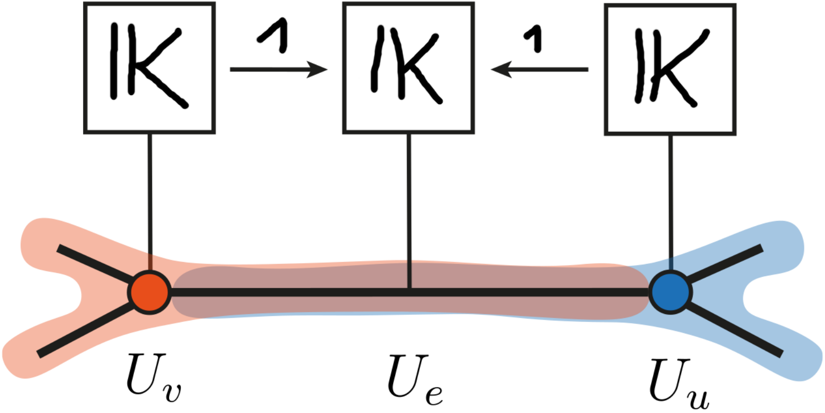
(ii) In this case we need nothing more **A little miracle**

- Here is an example:



- **Constant sheaf** = all vector spaces are the same, all maps identities

Structure sheaf



- Structure sheaf = all vector spaces are the field, all maps are identities
- Boils down to: Every vertex has the field, say $\mathbb{K}$, every edge-mid-point the field as well, and every interval between vertex and edge-mid-point gets the scalar 1

Thank you for your attention!

I hope that was of some help.