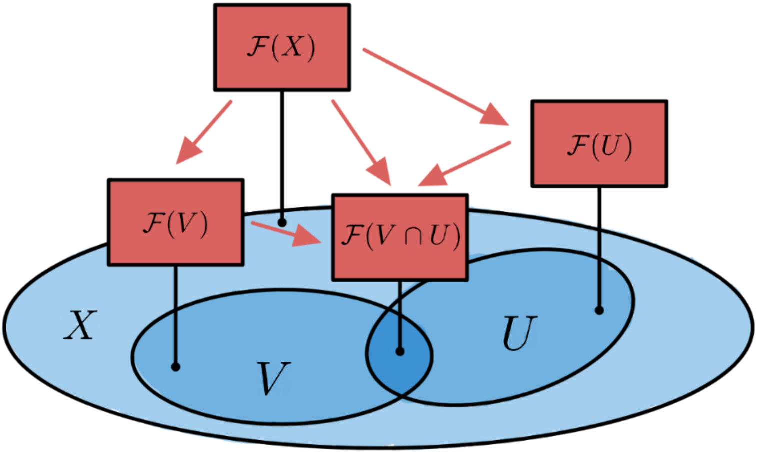
What are...sheaves, take 3?

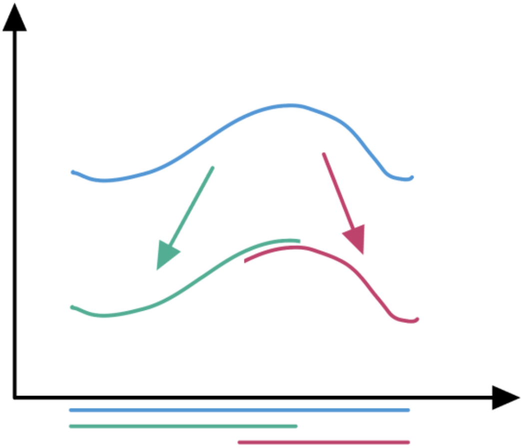
Or: Big from small

Presheaf of continuous functions $\mathcal{F}_c$



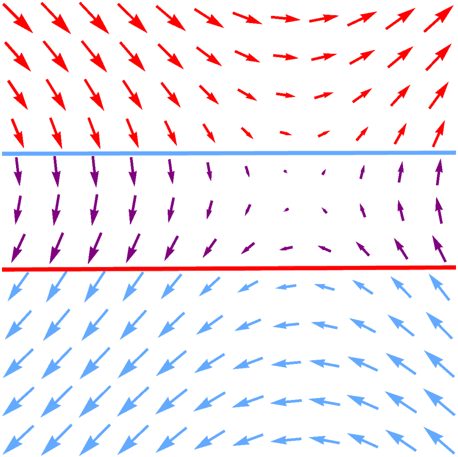
- Take the real line $\mathbb{R}$ as our underlying space
- $\mathcal{F}(U) = \{f : U \to \mathbb{R} | f \text{ is continuous}\}$ for all open $U \subset \mathbb{R}$
- We have (honest) restriction maps $\mathcal{F}(U,V) : \mathcal{F}(U) \to \mathcal{F}(V)$ for $V \subset U$

Locality



- $\mathcal{F}_c$ satisfies the following locality property
- $f \in \mathcal{F}(U)$ is determined by what it does on an open cover $(U_i)_{i \in I}$
- (L) For any $f, g \in \mathcal{F}(U)$, if $f|_{U_i} = g|_{U_i}$ for all i, then $f = g$

Gluing



- $\mathcal{F}_c$ satisfies the following gluing property
- $f \in \mathcal{F}(U)$ can be glued together from $(f_i \in \mathcal{F}(U_i))_{i\in I}$ for an open cover $(U_i)_{i\in I}$
- (G) If $f_i|_{U_i\cap U_j} = f_j|_{U_i\cap U_j}$ for all i, j, then there is a unique f with $f|_{U_i} = f_i$ for all i

For completeness: A formal statement

A presheaf $\mathcal{F}$ (of sets) on a topological space X consists of:

(i) A set $\mathcal{F}(U)$ for all $U \subset X$ open; $f \in \mathcal{F}(U)$ are called sections (of $\mathcal{F}$ over U)

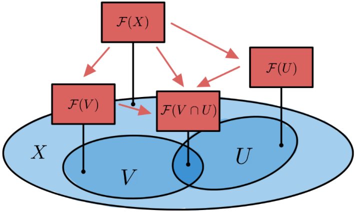
(ii) For $V \subset U$ a function ("restriction") $\mathcal{F}(U,V)\colon \mathcal{F}(U) \to \mathcal{F}(V)$ with $\mathcal{F}(U,U) = id$ with: if $W \subset V \subset U$, then $\mathcal{F}(U,W) = \mathcal{F}(V,W) \circ \mathcal{F}(U,V)$

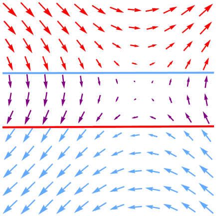
A sheaf $\mathcal{F}$ is a presheaf with:

(I) Locality (L) as on the previous slides ($f|_U = \mathcal{F}(X,U)(f)$)

(II) Gluing (G) as on the previous slides

- Presheaf: a collection of spaces related by restriction; sheaf: big from small

Presheaf: 

, sheaf: 

- Sheaf example Continuous functions; Non-sheaf example Bounded functions

Sheaf in algebraic geometry (AG)

- AG = the study of varieties using algebra
- Sheaves in AG: Over varieties with Zariski topology, enriched in rings or algebras (replace set by ring etc.)

Thank you for your attention!

I hope that was of some help.