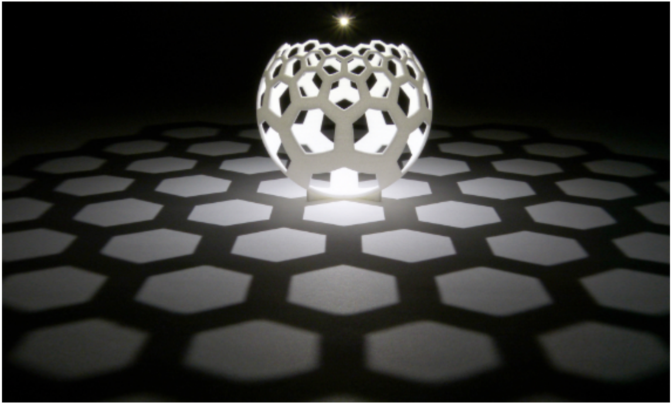
What are...prevarieties?

Or: Patchworks

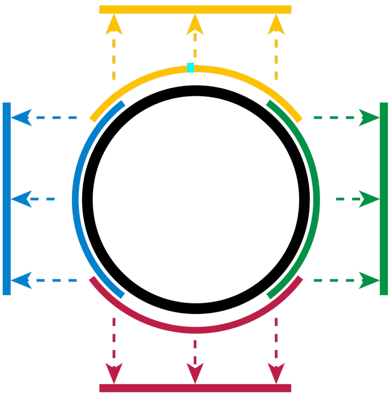
Varieties

The sphere is $\mathbb{R}^2$ plus a point at ∞ :



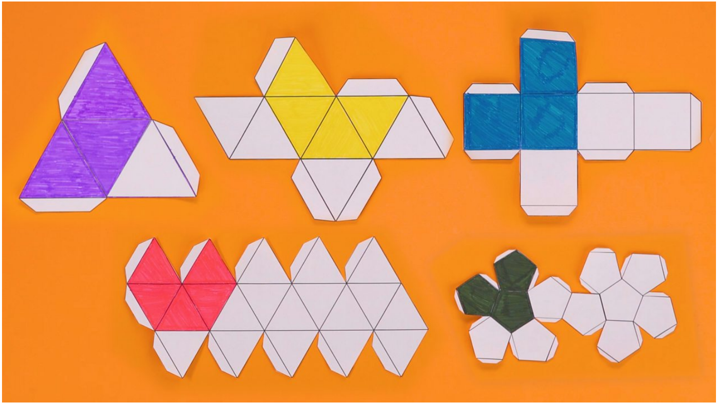
- Affine varieties are the main object of study of AG
- Mild problem They are almost never bounded hence almost never compact
- Fix At points at infinity in some controlled way

Patchworks aka manifolds



- Manifold $=$ something that is patched together from 'discs'
- Key for us Manifolds are often compact, but locally they resemble $\mathbb{R}^n$
- Idea Mimic that in AG!

Gluing



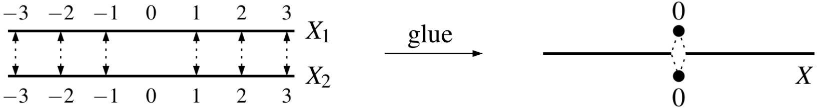
- Gluing along open sets is an important tool in topology
- We can do the same in AG (using the Zariski topology)
- Mild technicality We need to address gluing for ringed spaces

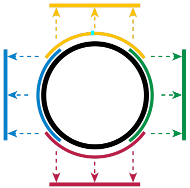
For completeness: A formal statement

Define:

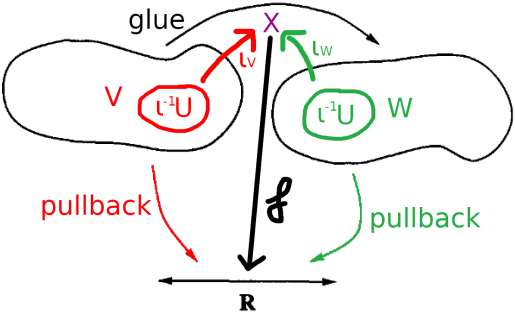
- A prevariety is a ringed space with a finite open cover by affine varieties
- Morphisms of these are morphisms of ringed spaces

- Every (open subset of an) affine variety is a prevariety
- Gluing is compatible with ringed spaces (next slide) and we get many examples this way





Gluing ringed spaces



- Patching Say V is patched together from V_1, V_2
- Say V_1, V_2 are ringed spaces
- The V is a ringed space via

$$\mathcal{O}_V(U) = \{f\colon U \to \mathbb{K} | \iota_V^*(f) \in \mathcal{O}_V(\iota_V^{-1}(U)) \text{ and } \iota_W^*(f) \in \mathcal{O}_W(\iota_W^{-1}(U))\}$$

f is regular if it is regular when restricted to both patches

Thank you for your attention!

I hope that was of some help.