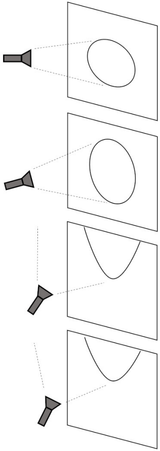
What are...conic sections?

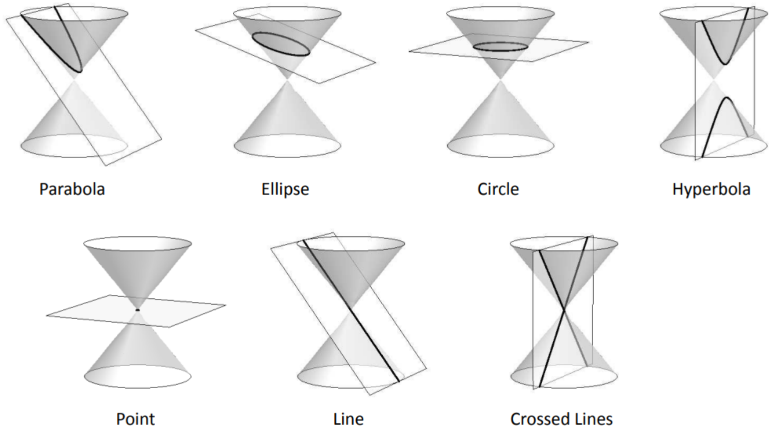
Or: Light cones and walls

Enter: the flashlight



- Conic section = whatever you get by pointing a flashlight towards a wall
- Three types: Ellipse (includes circle), parabola and hyperbola
- This is known since at least 300BC (except the flashlight part)

The more classical way



- Classical way = "How the old Greeks would have done it"
- Up to degenerate cases, conic sections are ellipse, parabola, hyperbola
- What does with have to do with algebraic geometry?

Degree two

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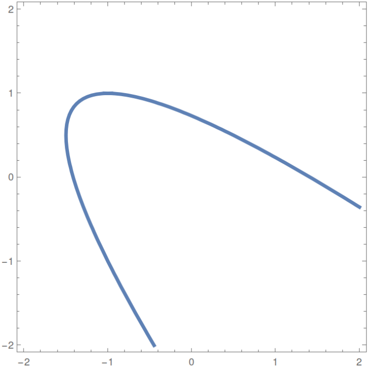
In[•]:= Conic[A_, B_, C_, D_, E_, F_] := ContourPlot[A*x^2 + B*x*y + C*y^2 + D*x + E*y + F == 0, {x, -2, 2},
          {y, -2, 2}, ContourStyle → Thickness[0.01]];

In[•]:= Conic[1/2, 1, 1/2, 0, 1, -1]
```

Out[•]=



- Generic equation of degree two

$$(*)\ A \cdot x^2 + B \cdot xy + C \cdot y^2 + D \cdot x + E \cdot y + F = 0$$

- Example The circle is $x^2 + y^2 - 1 = 0$, so $(A, B, C, D, E, F) = (1, 0, 1, 0, 0, -1)$

For completeness: A formal statement

The affine varieties of degree two are precisely the conic sections
(All of this up to degeneration)

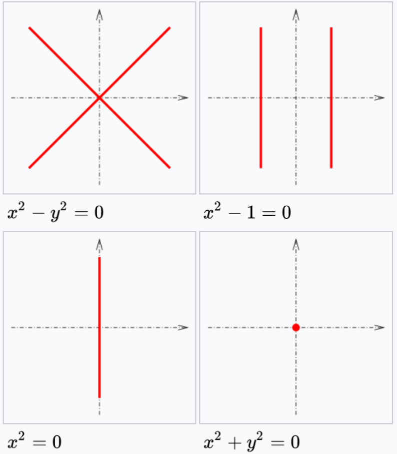
- The conic matrix:

$$M = \begin{pmatrix} A & B/2 & D/2 \\ B/2 & C & E/2 \\ D/2 & E/2 & F \end{pmatrix}$$

$$M_{12} = \begin{pmatrix} A & B/2 \\ B/2 & C \end{pmatrix}$$

- Then $V(*)$ is degenerate if and only if $\det M = 0$
- If $\det M \neq 0$, then:
    - $V(*)$ is an ellipse if $\det M_{12} > 0$
    - $V(*)$ is a parabola if $\det M_{12} = 0$
    - $V(*)$ is a hyperbola if $\det M_{12} < 0$

The degenerate cases



- Sometimes the defining equation (*) factors
- Example $x^2 - y^2 = 0$ factors as $(x - y)(x + y) = 0$
- (*) factors over $\mathbb{C}$ if and only if $V(*)$ is degenerate

Thank you for your attention!

I hope that was of some help.