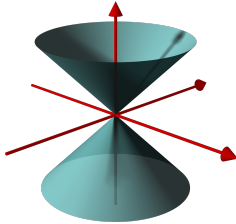**What is...a blow up, take 1?**
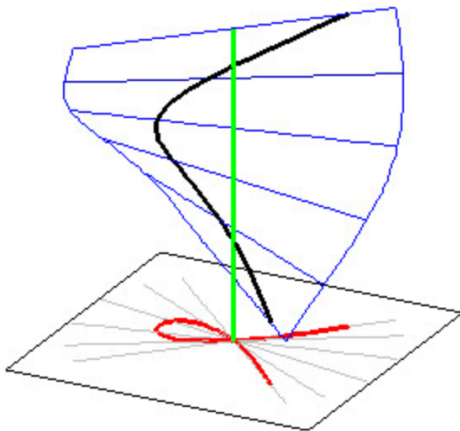
Or: Blowing up balloons

# Singularities are interesting (=good and bad)

- ▶ Singularities , in life and mathematics, are "point-events that change everything"

- ▶ The birth of a child drastically changes your life

- ▶ The most interesting part of a cone is the point at the origin

Blow up singularities

- ▶ Idea Study singularities by finding a nonsingular space birationally equivalent to the original one

- ▶ Why? Well, singular is interesting, but also difficult

Adding a twisted time direction
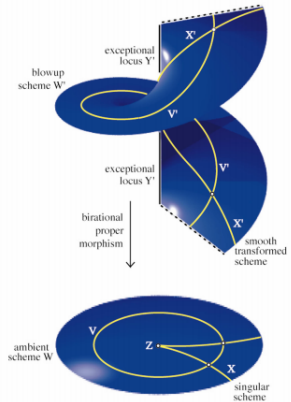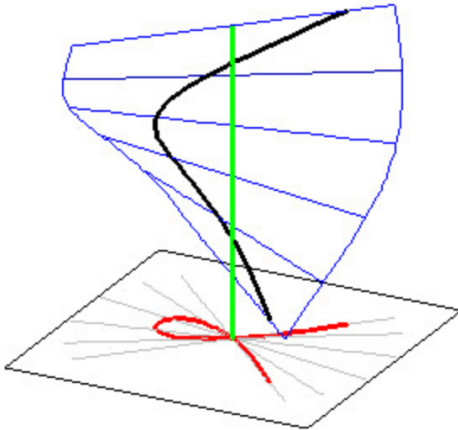
- ▶ Idea Remove singularities by "pulling them into space"

- ▶ Example (left) The double point becomes two points

- ▶ Example (right) Twisting the top, the cusp is removed as well

For completeness: A formal statement

For some affine variety $V \subset \mathbb{K}^n$ fix:

(i) Polynomials $f_1, ..., f_k \in \mathbb{K}[V]$

(ii) $U = V \setminus V(f_1, ..., f_k)$

(iii) The evaluation $f : U \to \mathbb{P}^{n-1}, x \mapsto (f_1(x) : ... : f_r(x))$

(iv) The graph $\Gamma_f = \{(x, f(x))\} \subset U \times \mathbb{P}^{n-1}$

The closure of Γ_f, denote $\tilde{V}$ is the blow-up of V at $f_1, ..., f_k$

- ▶ f is well-defined since $f_1, ..., f_k$ do not vanish simultaneously
- ▶ Projective space does the twist automatically
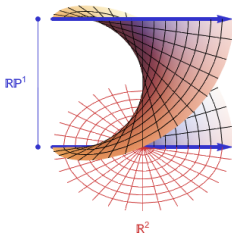
Nomenclature

- ▶ The name comes from inflation like "blowing up a balloon"

- ▶ Alternatively Read blow up as "zooming in on a photograph"

- ▶ This is not meant in the sense of explosion

Thank you for your attention!

I hope that was of some help.