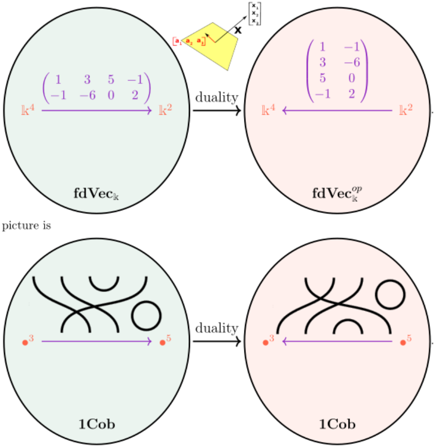
What are...dual curves?

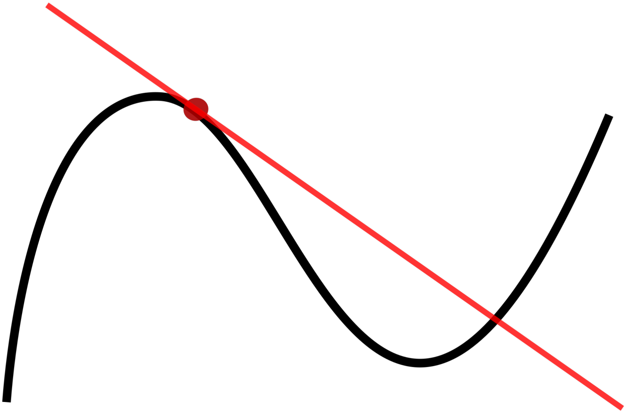
Or: Ellipse stays ellipse

Duality in mathematics



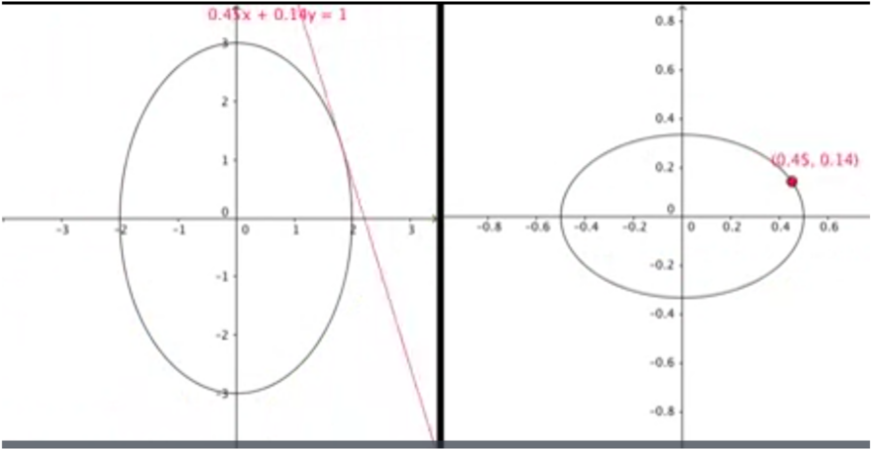
- Duality is everywhere in mathematics
- Example Dual vector space
- Today Duals of curves

Many tangents



- Setup Take a reasonably smooth function and consider tangents on it
- Observation The tangent varies smoothly when moving the point
- Question Is the set of tangents again a nice space?

Ellipse stays ellipse



- Above, left: The ellipse V given by $(x/2)^2 + (y/3)^2 = 1$
- Above, right: The ellipse V^* given by $(2x)^2 + (3y)^2 = 1$
- Observation: The tangents of V form V^*

For completeness: A formal statement

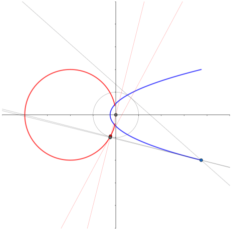
The dual V^* of a curve $V = V(f = 0) \subset \mathbb{P}^2$ is:

(i) Say $f(x, y, z) = 0$, assume $\mathbb{K}$ is not of characteristic two and the partial derivatives of f do not vanish simultaneously

(ii) Let $D\colon V \to \mathbb{P}^2, a \mapsto (\partial f/\partial x(a), \partial f/\partial y(a), \partial f/\partial z(a))$

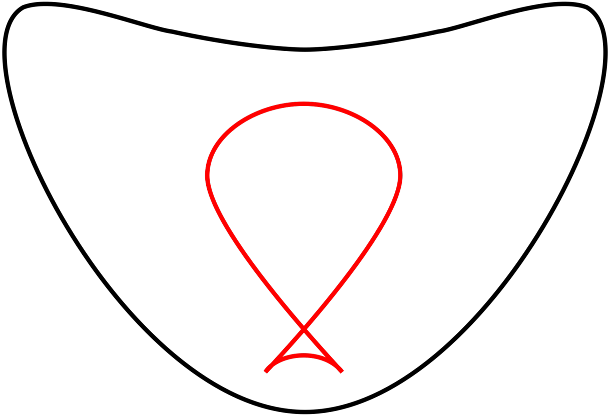
(iii) The image of D is V^*

- The dual of a conic is a conic



- Example The dual of $ax^2 + by^2 + cz^2 = 0$ is $\frac{1}{a}x^2 + \frac{1}{b}y^2 + \frac{1}{c}z^2 = 0$

The dual is almost perfect



- Fact The dual curve is a curve
- Fact If V is smooth and convex, then so is V^*
- Above The crossing comes from the two top extreme points; the cusps from the inflection points

Thank you for your attention!

I hope that was of some help.