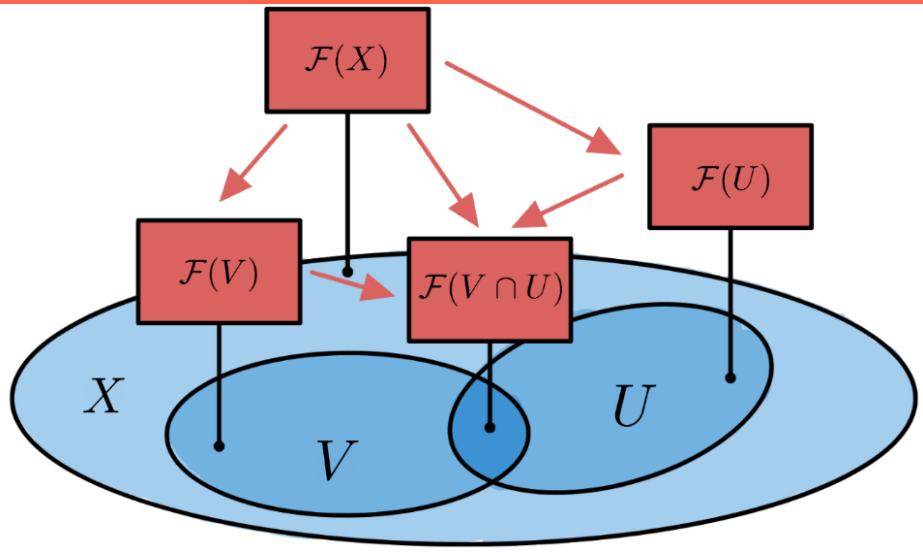
What are...sheaves of modules?

Or: The weather on earth

Reminder on sheaves



Sheaves, in a limited way for now and only on some (complex) manifold M:

(i) A presheaf $\mathcal{F}$ is a collection

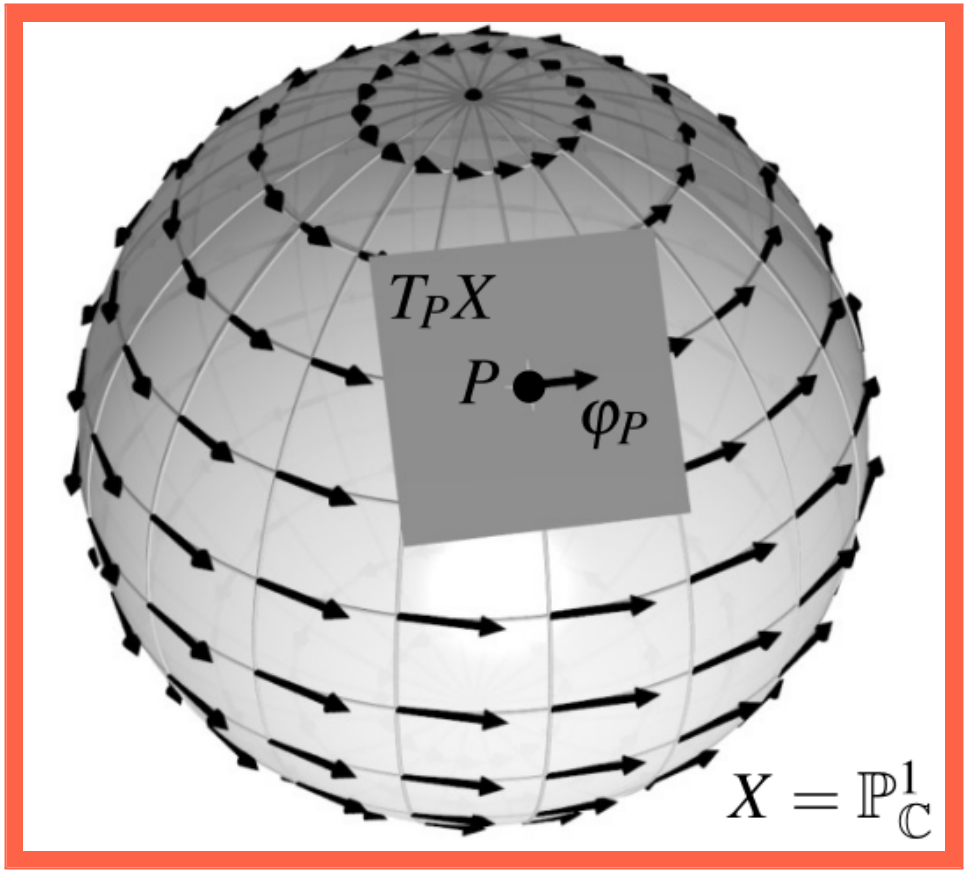
$$\mathcal{F}(U) = \{f : U \to \mathbb{C} \text{ satisfying condition X (e.g. holomorphic)}\}$$

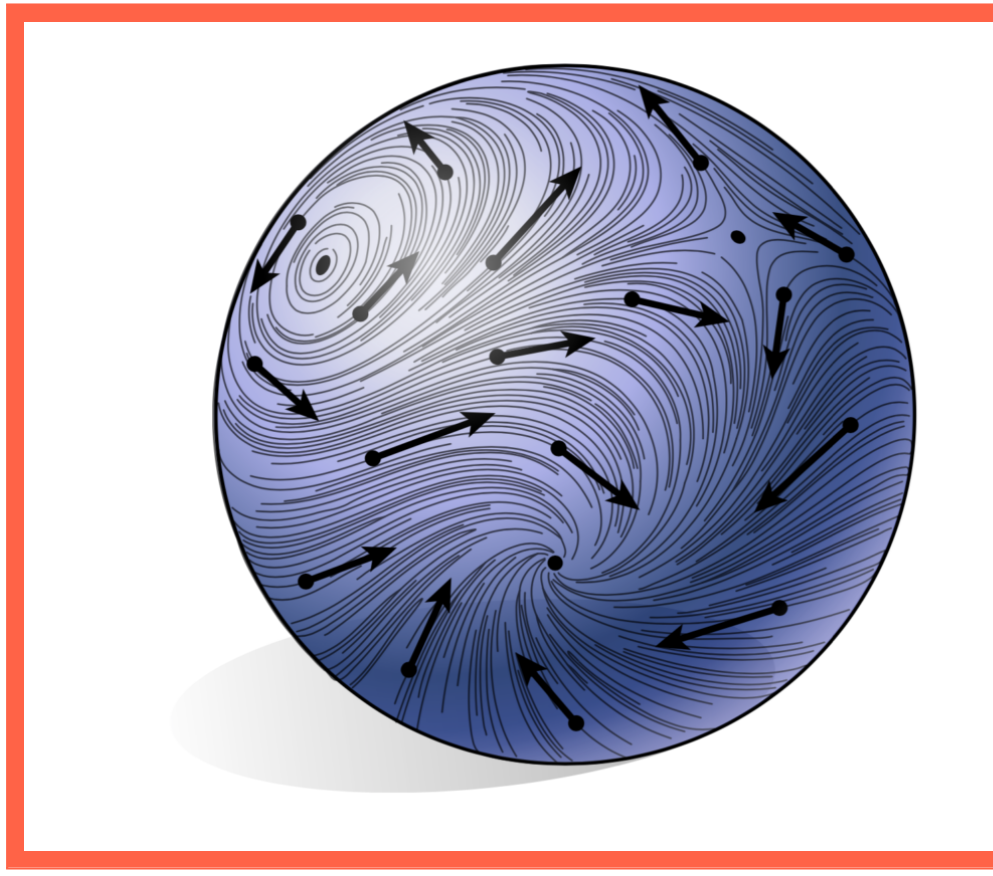
for any nonempty open subset $U \subset M$ such that the restriction $f|_{U'} \in \mathcal{F}(U')$ for all $f \in \mathcal{F}(U)$ and $U' \subset U$

(ii) A presheaf $\mathcal{F}$ is a sheaf if for any open set $U \subset M$ and any open cover U_i: $f \in \mathcal{F}(U)$ if $f|_{U_i} \in \mathcal{F}(U_i)$

- Above Reminder on sheaves (of rings)
- So far we did this with attaching rings (of regular functions) to open sets
- Today Not all examples fit into this framework!

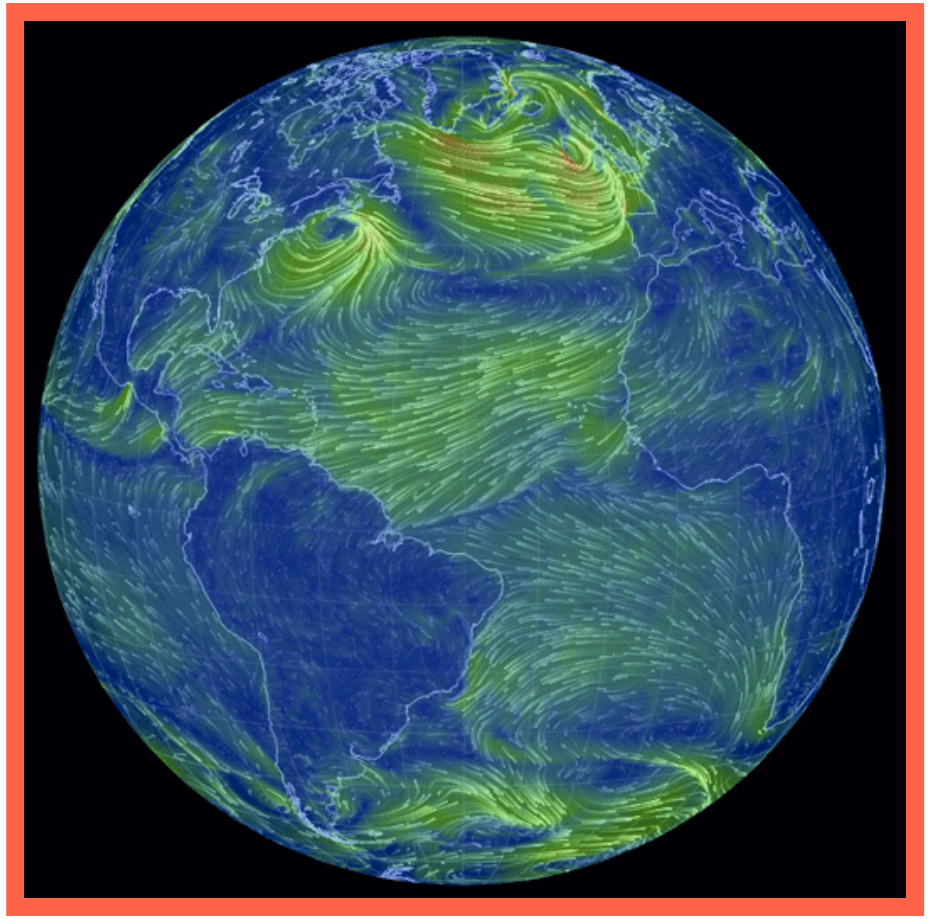
Tangent vector fields





- For a smooth curve V we had the tangent (space) $T(V)$ as a 1d vector space
- Maybe construct a tangent sheaf $\mathcal{T}_V$ such that a section over U (open) is given by specifying a tangent vector to V at all points which "vary nicely"
- This is also often called a tangent vector field

This is a key example!



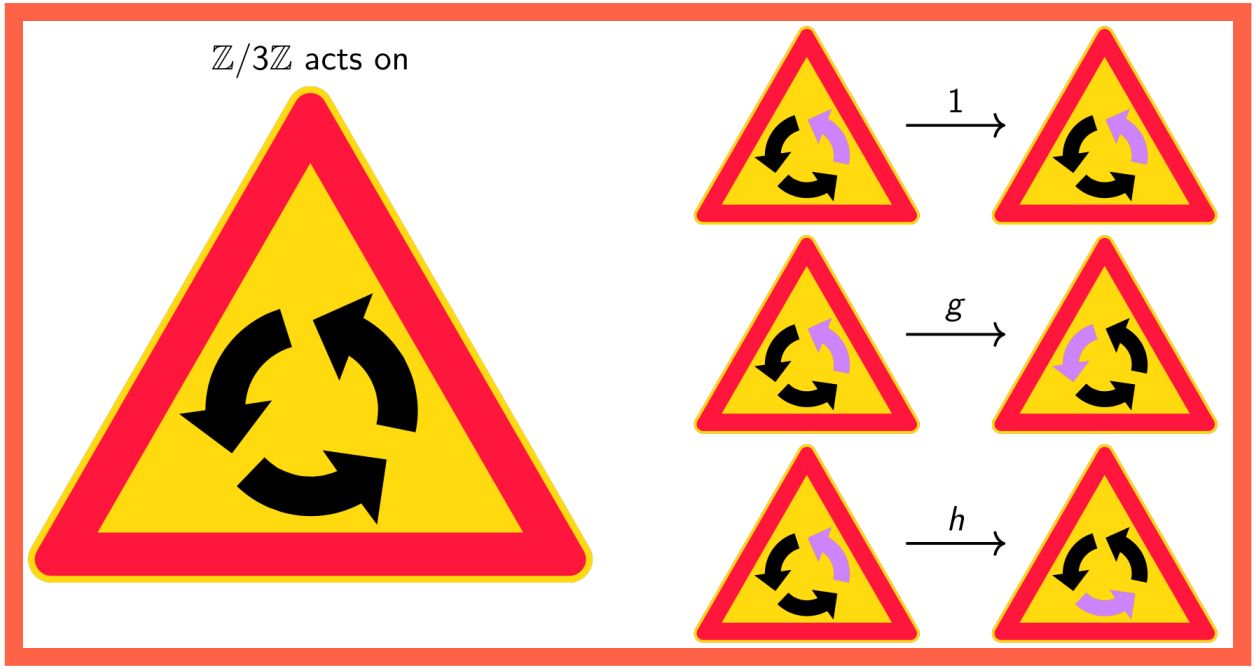
- Weather on earth is a tangent sheaf on $\mathbb{P}^1 \cong S^2$
- Tangent vector fields are important examples beyond algebraic geometry
- Problem We cannot multiply tangent vectors $\Rightarrow$ not a sheaf of rings

For completeness: A formal statement

(Pre)sheaf of $\mathcal{O}_V$-modules $=$ a (pre)sheaf $\mathcal{F}$ such that:

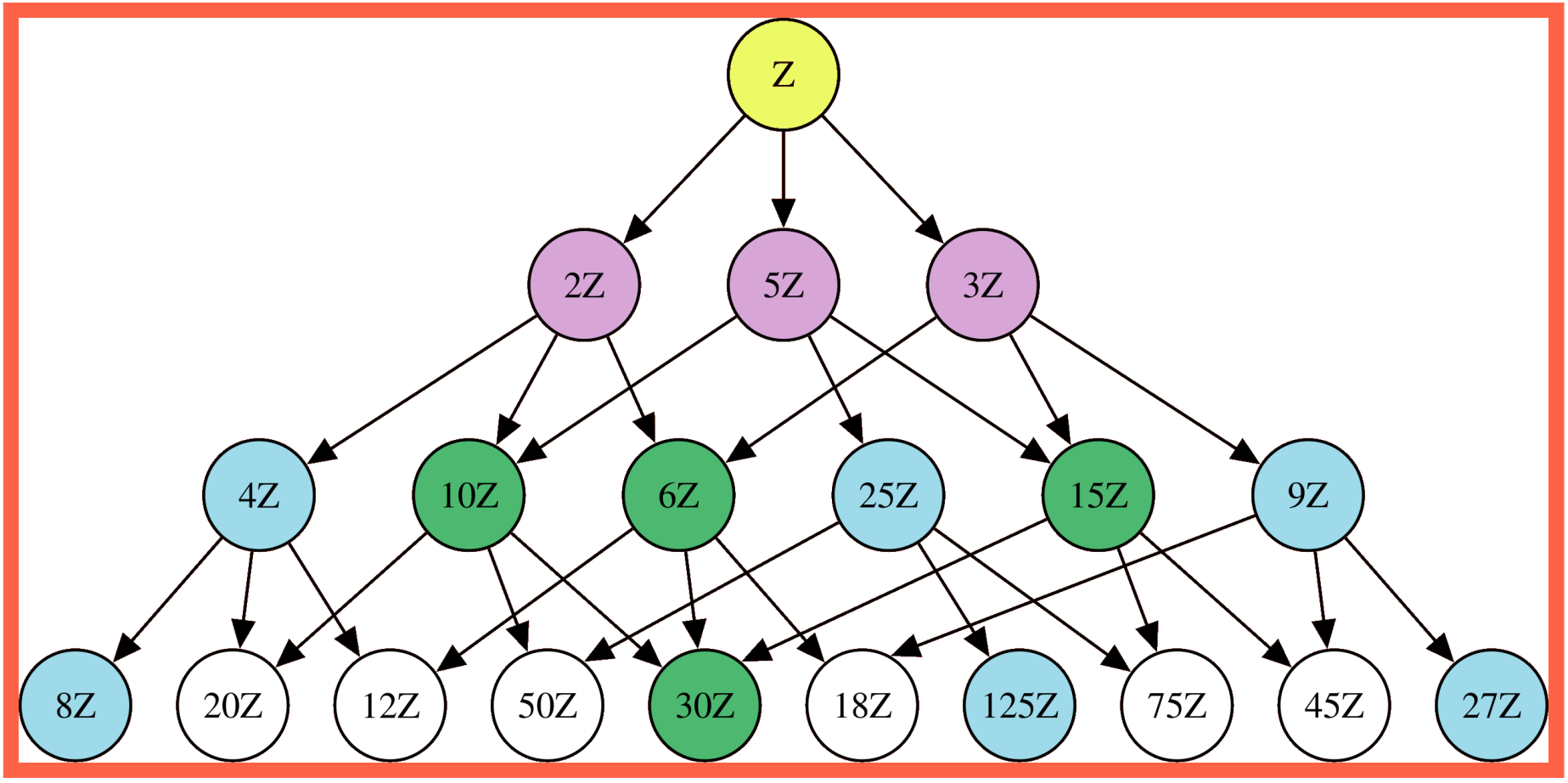
- $\mathcal{F}(U)$ is an $\mathcal{O}_V$-module for all open subsets $U \subset V$
- All restriction maps are $\mathcal{O}_V$-module maps

- Recall a module/representation (for groups, rings, algebras...):



- Recall $\mathcal{O}_V$ = ring of regular functions

Sheaf of ideals



- Observations Ideals are modules
- Recall Ideal $I \subset R$ if $rir' \in I$ for $r, r' \in R$, $i \in I$
- Sheaves of ideals are other examples of sheaves of modules

Thank you for your attention!

I hope that was of some help.