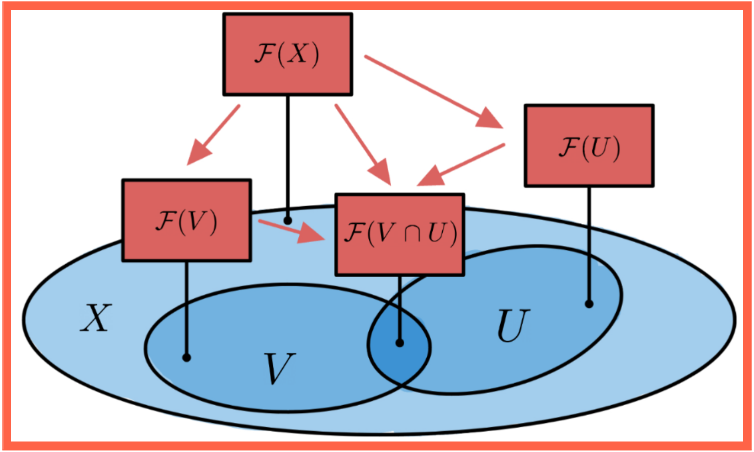
What is...a skyscraper sheaf?

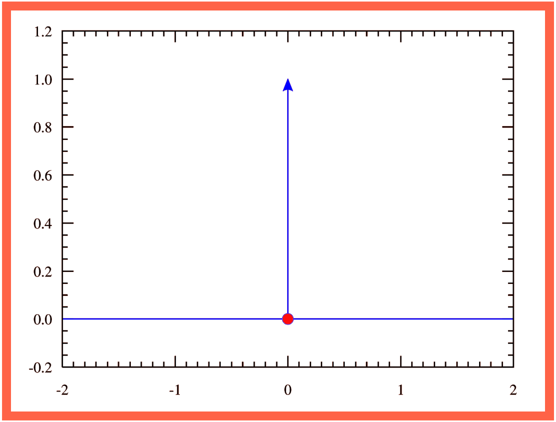
Or: Dirac's delta

Sheaves of modules



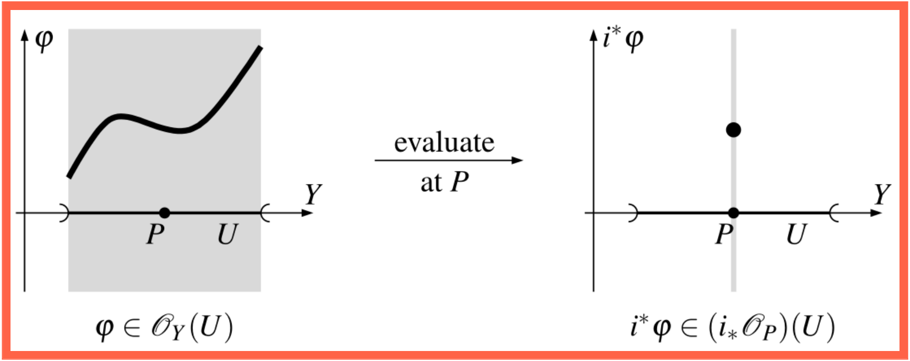
- Recall Sheaves of modules have $\mathcal{O}_V$-modules attached to open sets
- Essentially this means that they have vector spaces attached to open sets
- Observation The most linear-algebra-type constructions should work!

Indicator functions



- Indicator functions (of a point) takes value 1 (or similar) at a point and vanishes everywhere else
- Example Dirac's delta
- The point These are very useful but also very easy

Make it a sheaf!



- **Setting** Let p be a point of V over $\mathbb{K}$, set

$$\mathbb{K}_p(U) = \begin{cases} \mathbb{K} & \text{if } p \in U, \\ 0 & \text{if } p \notin U, \end{cases} \text{ for all open } U \subset V$$

- **Sections** = functions on V that only have a value at p
- $\mathbb{K}_p$ is called **skyscraper sheaf**

For completeness: A formal statement

$\mathbb{K}_p$ is a sheaf

- The name comes from thinking about a line connecting p and $f(p)$

- Alternatively, this can be construction by push forward ι_* of sheaves (a linear algebra construction) along the inclusion $\iota\colon p \to V$
- There is also the pull back of sheaves ι^* (another linear algebra construction)

More linear algebra

$$\begin{pmatrix}1\\2\\3\end{pmatrix} \otimes \begin{pmatrix}4\\5\end{pmatrix}$$

$\otimes$	4	5
1	$1 \cdot 4$	$1 \cdot 5$
2	$2 \cdot 4$	$2 \cdot 5$
3	$3 \cdot 4$	$3 \cdot 5$

- These give sheaves Direct sum and tensor product
- More sheaves Kernel and image
- There is no issue to extend these linear algebra definitions to sheaves

Thank you for your attention!

I hope that was of some help.