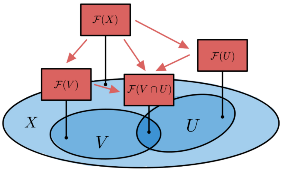
What is...sheafification?

Or: Freeness

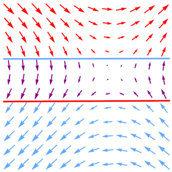
Presheaves and sheaves

- Presheaf: a collection of spaces related by restriction; sheaf: big from small

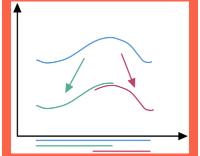
Presheaf:



, sheaf:

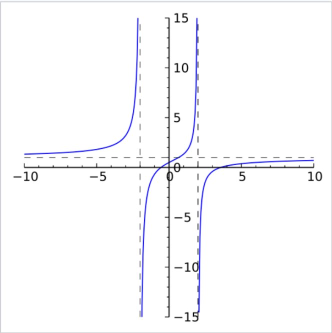


- Sheaf example Continuous functions; Non-sheaf example Bounded functions

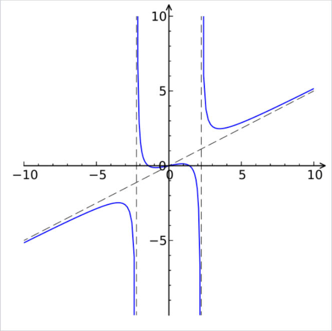


- Presheaf but nonsheaf examples Rational or continuous bounded functions
- These are nonlocal properties so they are not sheaves
- Question Is there a way to make them into sheaves?

Regular functions



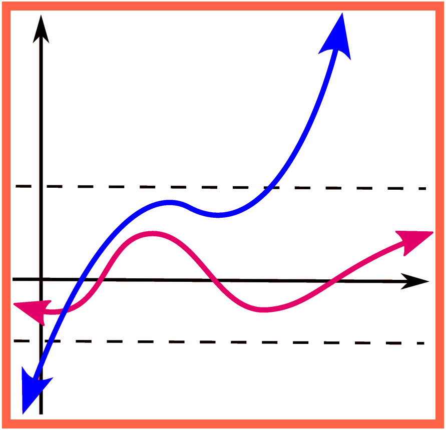
Rational function of degree 2, with a graph of degree 3: $y = \dfrac{x^2 - 3x - 2}{x^2 - 4}$



Rational function of degree 3, with a graph of degree 3: $y = \dfrac{x^3 - 2x}{2(x^2 - 5)}$

- **Regular functions** = functions of the form p/q for polynomials p, q
- **Local regular functions** = functions f so that al points have neighborhoods such that f is of the form p/q for polynomials p, q
- **Observation** One sits in the other

Bounded functions



- Bounded continuous functions $=$ continuous functions f of the form $|f| <$ some constant
- Continuous functions could be also e.g. x on $\mathbb{R}$
- Observation One sits in the other

For completeness: A formal statement

Sheafification (a sheaf "canonically" associated to a presheaf)

In the words of *The Stacks project*:

The basic construction is the following. Let $\mathcal{F}$ be a presheaf of sets on a topological space X. For every open $U \subset X$ we define

$$\mathcal{F}^{\#}(U) = \{(s_u) \in \prod_{u \in U} \mathcal{F}_u \text{ such that } (*)\}$$

where $(*)$ is the property:

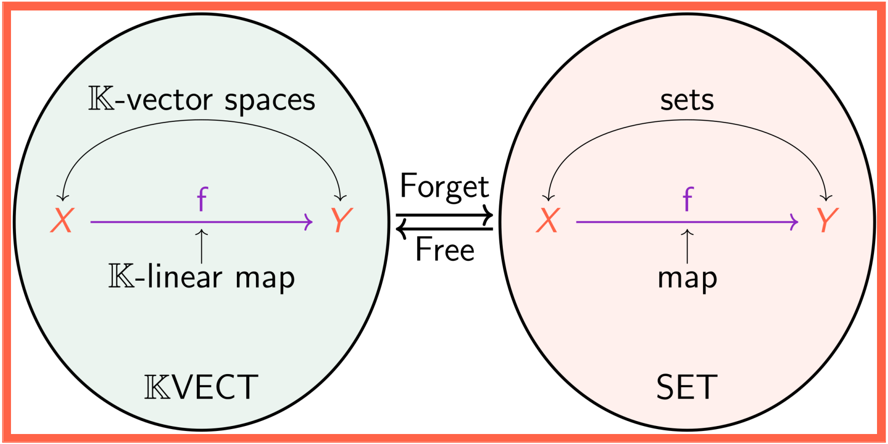
(*)
For every $u \in U$, there exists an open neighbourhood $u \in V \subset U$, and a section $\sigma \in \mathcal{F}(V)$ such that for all $v \in V$ we have $s_v = (V, \sigma)$ in $\mathcal{F}_v$.

The sheafification of rational functions are local rational functions; the sheafification of bounded continuous functions are continuous functions

- There is a "better" explanation on the next slide using category theory

From *Math with Bad Drawings* we get:

Free sheaves



- Free-forget adjunctions are everywhere in mathematics
- There is a forgetful functor Forget from sheaves to presheaves
- Sheafification $=$ left adjoint of Forget

Thank you for your attention!

I hope that was of some help.