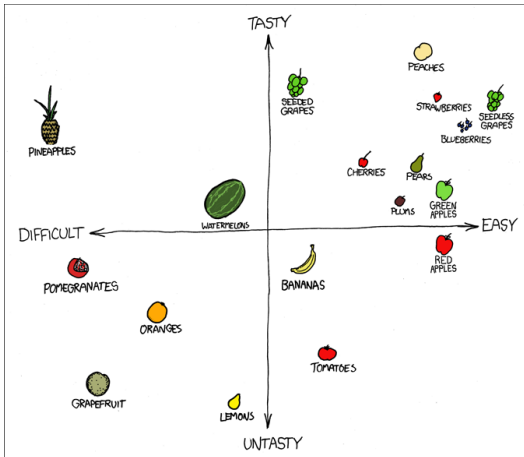


What are...graph polynomials?

Or: Easy problems, not hard problems

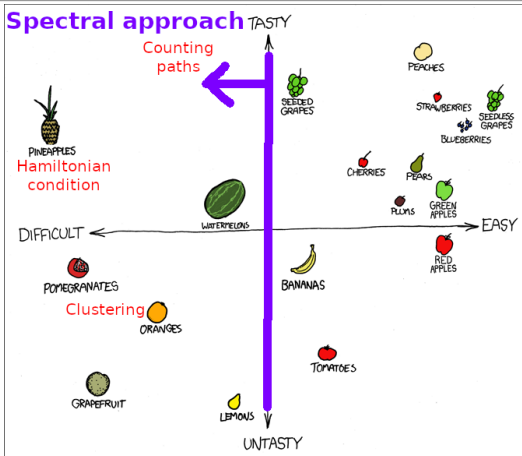
Easy and hard



Oversimplified and whatever that means, graph theory is divided into:

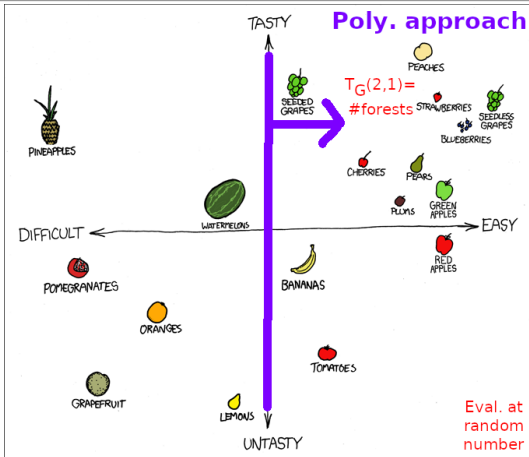
- ▶ Easy and difficult problems
- ▶ Tasty and untasty problems

Spectral approach



- ▶ So far we have seen the spectral approach
- ▶ This is designed to tackle difficult problems approximately
- ▶ Spectral graph theory usually gives bounds or asymptotics

Polynomial approach



- ▶ We will see the polynomial approach
- ▶ This is designed to tackle easy problems nicely
- ▶ Polynomials usually give short and sweet explanations why something is easy

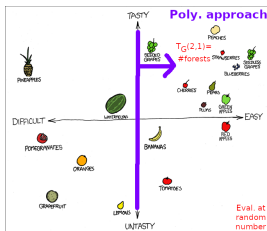
For completeness: A formal statement

There exists a polynomial $T_G(x, y)$ associated to a graph such that:

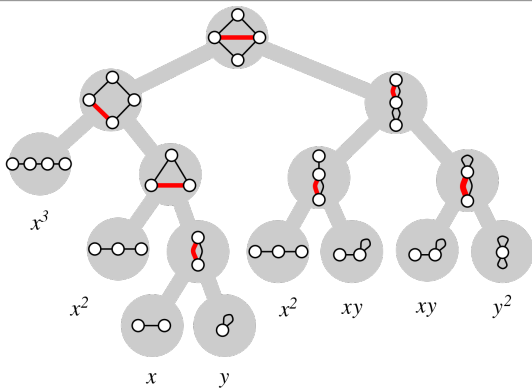
- ▶ $T_G(2, 1) = \#$ forests
- ▶ $T_G(1, 1) = \#$ spanning forests
- ▶ $T_G(1, 2) = \#$ spanning subgraphs
- ▶ More...

▶ The polynomial is called Tutte polynomial

▶ This “immediately” shows that counting e.g. spanning forest is easy



It gets even better



► $T_G(x, y)$ can be defined by a recursive formula (above)

► $T_G(x, y)$ can be defined by a closed formula

► $T_G(x, y)$ can be defined using the Potts model

► More

Thank you for your attention!

I hope that was of some help.