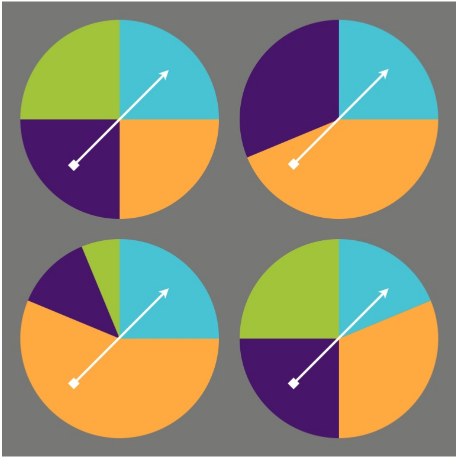
What are...random graph models?

Or: Different, yet the same

Top to bottom $G(n, M)$

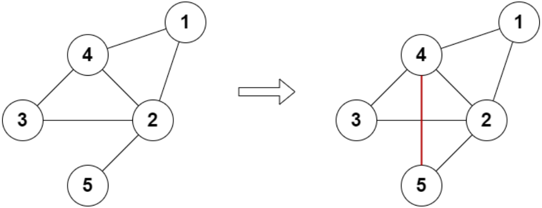


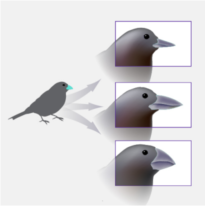
- Let us fix the number of vertices n, M
- Take the complete graph K_n and the set=bag of its subgraphs with M edges
- Every subgraph is equally likely drawn, by convention

Bottom to top $G_{n,p}$

- Let us fix the number of vertices n and a probability p
- Take the empty graph and run through pairs for vertices $v \neq w$
- Put an edge with probability p

Evolution $\mathcal{G}_n$





- Let us fix the number of vertices n
- $\mathcal{G}_n$ has sequences $G_0 \subset G_1 \subset \ldots$ of n vertex graph with subscript many edges
- Every sequence is equally likely chosen, by convention

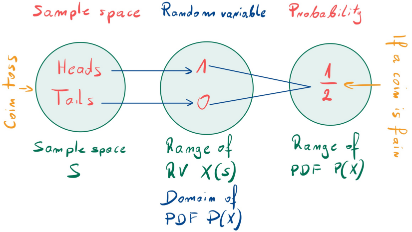
For completeness: A formal statement

For X_s=complete subgraphs of size s we have expectations (here $N = \binom{n}{2}, S = \binom{s}{2}$):

(1) $\mathbb{E}_p(X_s) = \binom{n}{s} p^S$ Finding complete graphs in $G_{n,p}$

(2) $\mathbb{E}_M(X_s) = \binom{n}{s}\binom{N-S}{M-S}\binom{N}{M}^{-1}$ Finding complete graphs in $G(n, M)$

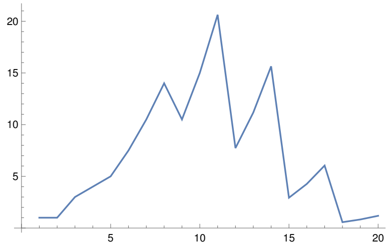
- $\mathbb{E}_p$ = expectation on $G_{n,p}$; $\mathbb{E}_M$ = expectation on $G(n, M)$
- Every graph invariant on a random graph space becomes a random variable



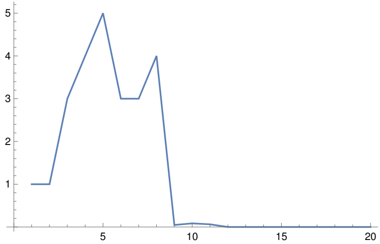
- The nature of such a random variable depends crucially on the space; well...

Different and equal answers

$$\frac{\mathbb{E}_{p=1/2}(X_{n/3})}{\mathbb{E}_{M=n(n-1)/4}(X_{n/3})}:$$



$$\frac{\mathbb{E}_{p=1/(n-1)}(X_{n/3})}{\mathbb{E}_{M=n/2}(X_{n/3})}:$$



- The three random graph models are somewhat different
- The three random graph models are somewhat the same

Thank you for your attention!

I hope that was of some help.