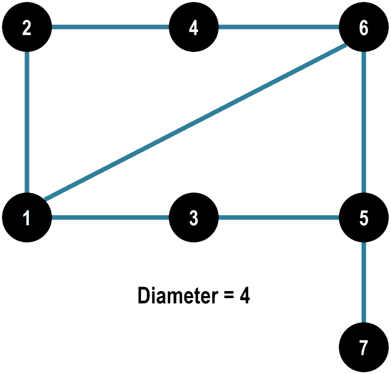
What is...the diameter of random graphs?

Or: The same diameter!?

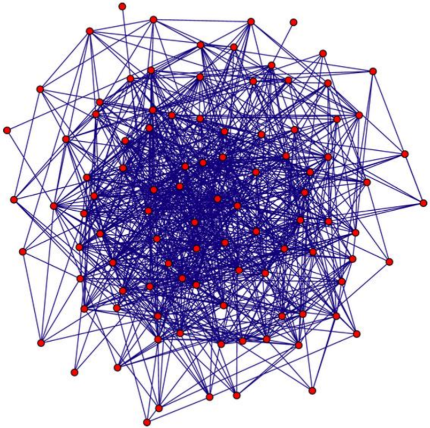
Diameter $d(G)$ of a graph G



v	1	2	3	4	5	6	7
1	0	1	1	2	2	1	3
2	1	0	2	1	3	2	4
3	1	2	0	3	1	2	2
4	2	1	3	0	2	1	3
5	2	3	1	2	0	1	1
6	1	2	2	1	1	0	2
7	3	4	2	3	1	2	0

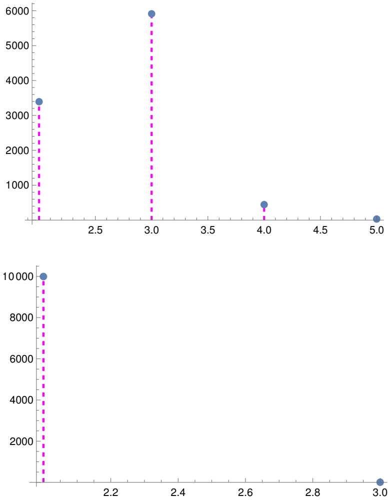
- $d(G)$ = length of the shortest path between the most distanced vertices
- $d(G)$ = how far we must travel from one end of G to the other
- $d(G) = \infty$ for non-connected graphs but we ignore that case

Many edges



- Recall that random graphs have many edges
- Expectation The diameter of almost all graphs is tiny
- Question How tiny? Certainly > 1 (only K_n has $d(G) = 1$). 2? 3? Bounded?

Testing diameters of random graphs



- Top The diameters of 10000 random coin flip graphs with 10 vertices
- Bottom The diameters of 10000 random coin flip graphs with 50 vertices

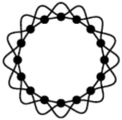
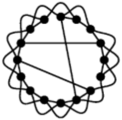
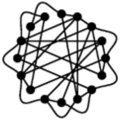
For completeness: A formal statement

Suppose $0 < p \leq 1$ and M are constant, then:

- Almost all $G_{n,p}$ have $d(G_{n,p}) = 2$
- Almost all $G(n, M)$ have $d\big(G(n, M)\big) = 2$

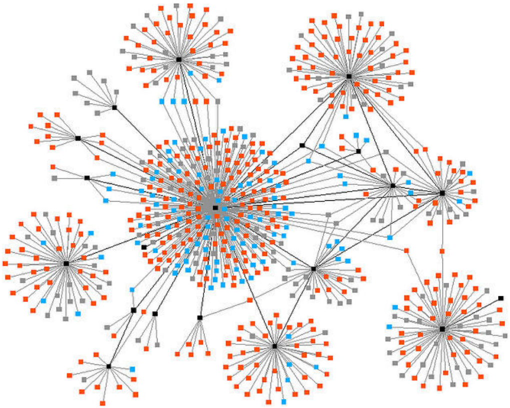
Hence, almost all graphs are tiny

- Even better, almost all graphs are equally tiny but not small world (up next)

			
Network	Lattice, Ordered	Small World	Random, Disordered
Clustering Coefficient	High	High	Low
Mean Path Length	Long	Short	Short

- There is also a statement for varying p and M

Small world is not quite random



- Small world ≈ networks like social media have small diameter
- It was quickly realized that small world needs different random graph models
- Problem The random graph models we have seen have no clusters

Thank you for your attention!

I hope that was of some help.