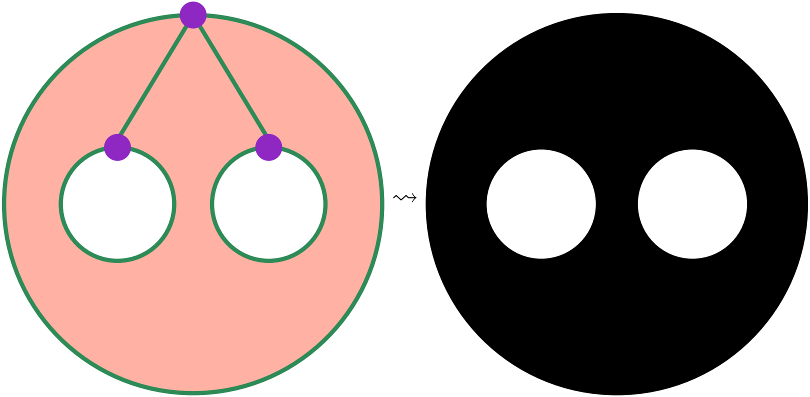
What are...operations on cell complexes?

Or: Cell by cell

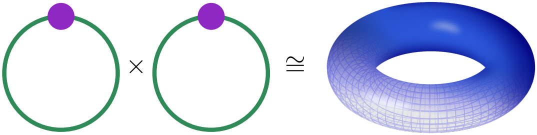
The Lego principle



- Cell complexes form a big class of examples Flexible
- Cell complexes allow cell-by-cell arguments Rigid

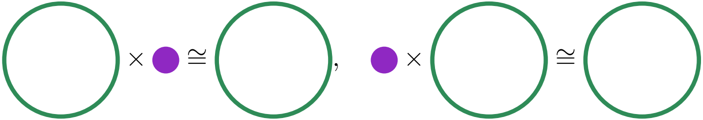
Question. How to construct new cell complexes from known ones?

Products of cell complexes

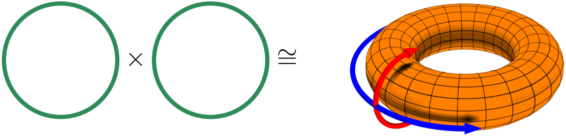
$S^1 \times S^1 =$  $=$ torus

- One 0-cell:

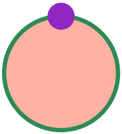
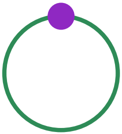
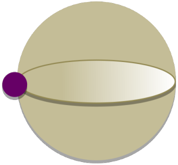
- Two 1-cells:



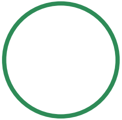
- One 2-cell:



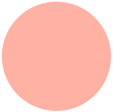
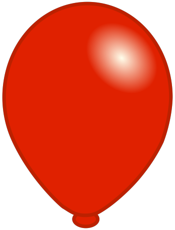
Quotient of cell complexes

$D^2/S^1 =$  $/$  $\cong$  $=$ sphere

- One 0-cell:

 and  $\rightsquigarrow$ 

- One 2-cell:

 $\rightsquigarrow$ 

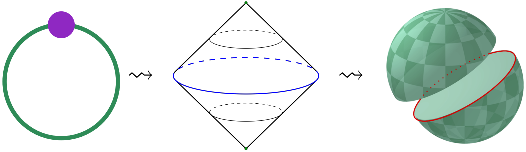
For completeness: A formal statement

Cell complexes are closed under the operation of...

(a) ...taking products (some care in needs to be taken in the infinite case)

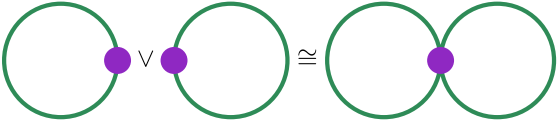
(b) ...taking quotients

(c) ...taking suspensions, *e.g.*



Similarly for joins

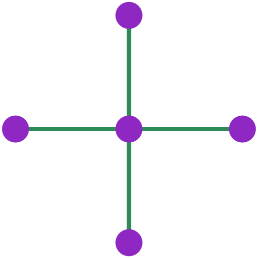
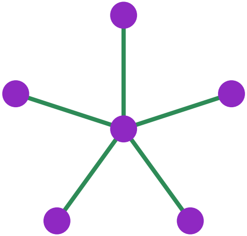
(d) ...taking wedge sums $\vee$, *e.g.*



(e) ...taking smash products $\wedge$ (via $\vee$ and quotients)

Stars a.k.a. careful with infinities

The star X_I with $\#I$-number of rays

$\#I = 4$: , $\#I = 5$: 

- The stars X_I are cell complexes
- The product $X_I \times X_J$ is a cell complex
- The weak topology of $X_I \times X_J$ does not need to be the product topology, *e.g.* if $I = \mathbb{R}$ and $J = \mathbb{N}$
- No problems in the finite case

Thank you for your attention!

I hope that was of some help.