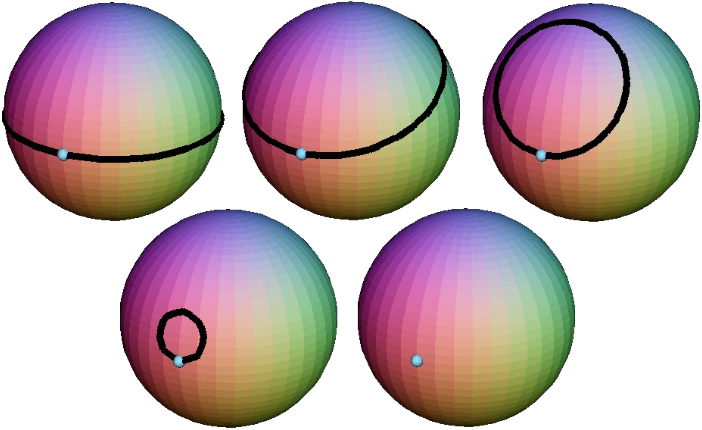
What is...cellular approximation?

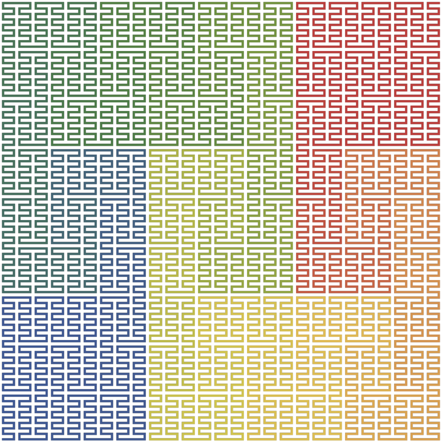
Or: I miss my space-filling curves...

Avoiding a point...



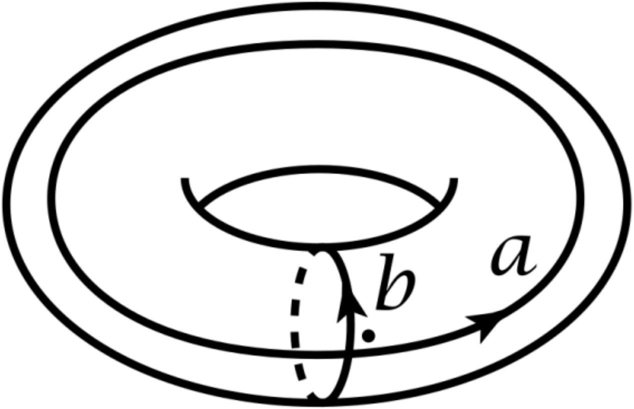
- Goal Show $\pi_k(S^n) \cong 0$ for $k < n$
- Strategy Poke a hole into S^n and contract the rest along with $S^k \to S^n$
- Catch Need to show that any $S^k \to S^n$ misses a point

...is not all that obvious



- Beware Space-filling curves exist
- We can still avoid a point
- "We can avoid a point" is a quite general statement

The torus with a fixed cell structure



$$T = T^0 \cup \underbrace{T^1}_{\bullet, a, b} \cup T^2$$

- Most maps $S^1 \to T$ are not contained in the 1-skeleton T^1
- But all maps $S^1 \to T$ are homotopic to either $\bullet$, a, b

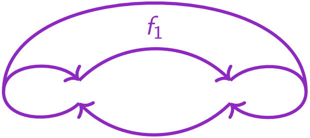
For completeness: A formal statement

A map $f: X \to Y$ between cell complexes is called cellular if

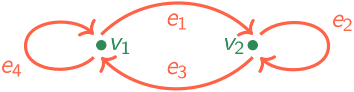
$$f(k\text{-skeleton}) \subset k\text{-skeleton} \quad \forall k$$

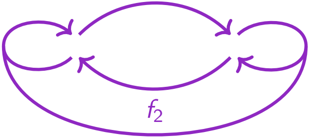
Every map between cell complex is homotopic to a cellular map

This works for any choice of cell structures!



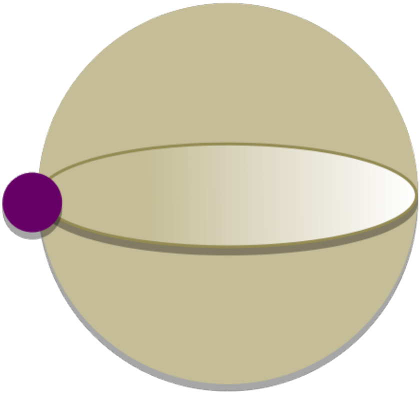
Also a torus:





Back to $\pi_k(S^n) \cong 0$ for $k < n$

$S^2 \leftrightsquigarrow$



- Take the balloon cell structure on S^n One 0- and one n-cell
- $S^k \to S^n$ can be assumed to end in the k-skeleton of S^n
- The k-skeleton of S^n is trivial for $k < n$ Done!

Thank you for your attention!

I hope that was of some help.