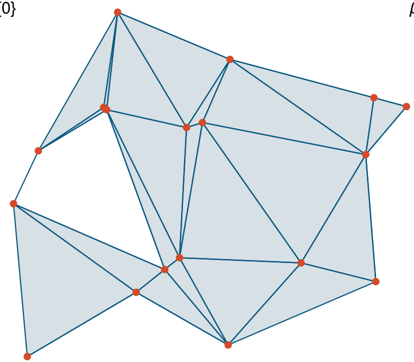
What is...persistent homology?

Or: Applications 2 (topology in data analysis)

Growing discs and homology

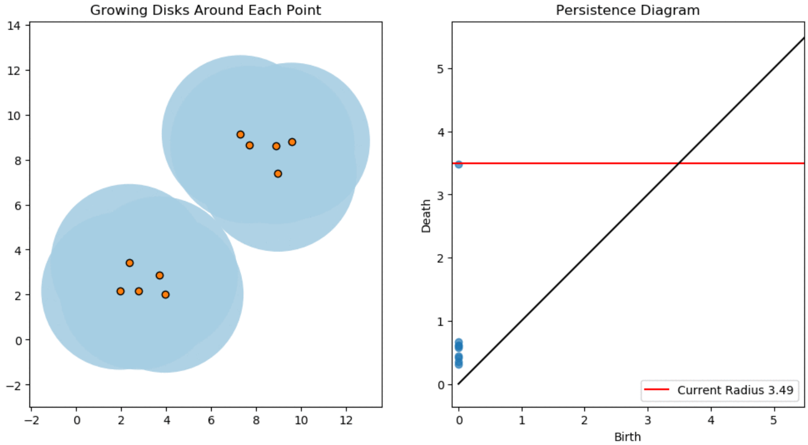
$H_0 \cong \mathbb{Z}$
$H_1 \cong \mathbb{Z}$
$H_2 \cong \{0\}$

$\beta_0 = 1$
$\beta_1 = 1$
$\beta_2 = 0$



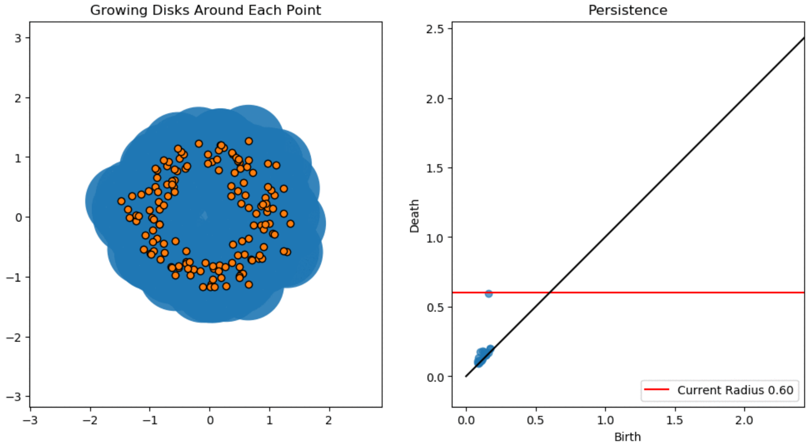
- As one increases a threshold, at what scale do we observe changes in data?
- There are many different flavors
- Today Discrete points in $\mathbb{R}^n$

0th persistent homology



- The 0th persistent homology measures how connected components change
- Birth New 0d holes=connected components (all born at zero at $y = x$)
- Death 0d holes=connected components vanish

1th persistent homology



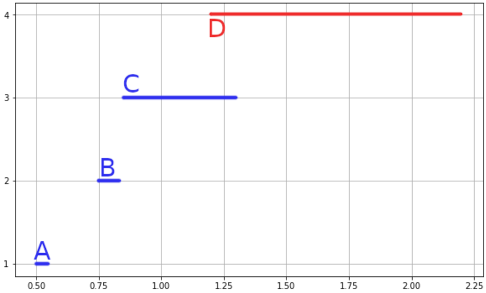
- The 1th persistent homology measures how internal circles change
- Birth New 1d holes=internal circles
- Death 1d holes=internal circles vanish

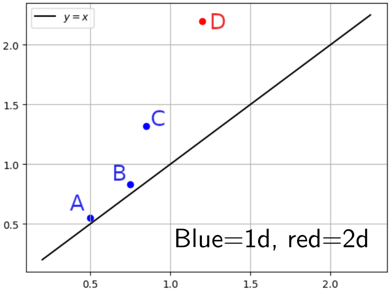
For completeness: A formal definition

X finite simplicial complex, $f : K \to \mathbb{R}$ with $f(\sigma) \le f(\tau)$ whenever σ is a face of τ

- $K(a) = f^{-1}(]-\infty, a])$ is a subcomplex, and we get $K_0 \subset ... \subset K_n = K$
- $K_i \hookrightarrow K_j$ for $i \le j$ induce $f_p^{i,j} \colon H_p(K_i) \to H_p(K_j)$
- pth persistent homology = images of these $f_p^{i,j}$

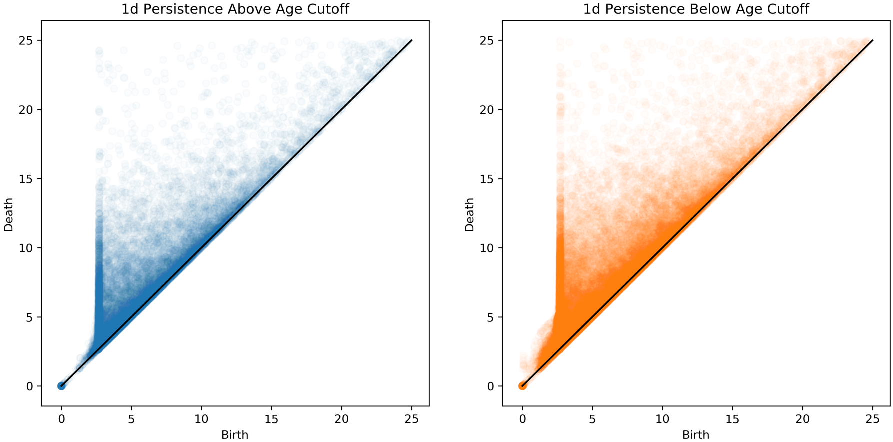
Persistence diagram Persistent nd holes are far-away from $y = x$





Example. C is born at 0.8 and dies at 1.3

Real-world applications of homology – one example



- **Homology** proved useful in detecting age differences in brain artery trees
- **Idea** Render brain artery trees into point-clouds and use persistent homology
- Differences are subtle – like most differences in human brains – but measurable

Thank you for your attention!

I hope that was of some help.