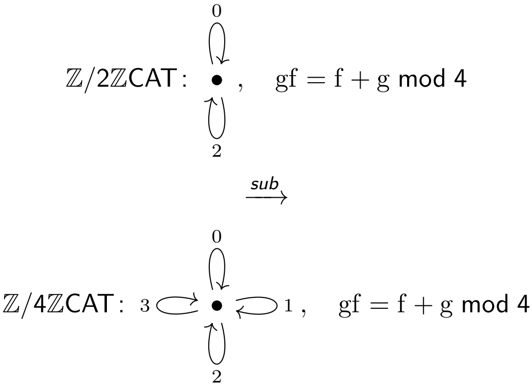
What is...a subcategory?

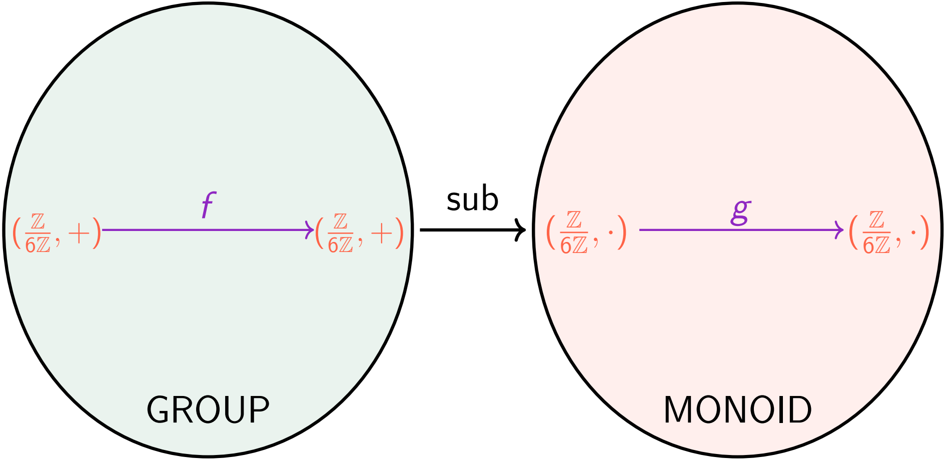
Or: The symmetric groups and cobordisms

Groups and groups



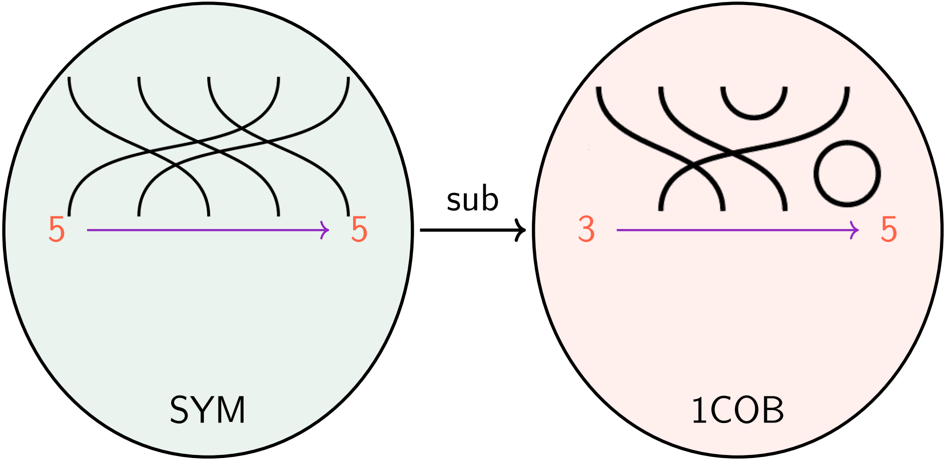
- G Objects •, arrows elements of G
- $\mathbb{Z}/2\mathbb{Z}$CAT is a substructure of $\mathbb{Z}/4\mathbb{Z}$CAT

Groups and monoids



- GROUP/MONOID Objects groups/monoids, arrows group/monoid homomorphism
- GROUP is a substructure of MONOID

Crossings in cobordisms



- SYM Objects $\mathbb{N}$, arrows 1:1 matching from bottom to top
- SYM is a substructure of 1COB

For completeness: A formal definition

C is called a subcategory of D if:

- Objects of $C \subset$ objects of D Sub on objects
- Arrows of $C \subset$ arrows of D Sub on arrows
- The relevant identities from D are in C
- The relevant compositions from D hold in C

- C is dense if objects of $C \cong$ objects of D

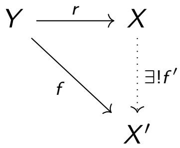
Dense:$\mathbb{Z}/2\mathbb{Z}\mathrm{CAT} \subset \mathbb{Z}/4\mathbb{Z}\mathrm{CAT}, \mathrm{SYM} \subset 1\mathrm{COB}$, not dense:$\mathrm{GROUP} \subset \mathrm{MONOID}$

- C is full if arrows of $C =$ arrows of D (whenever relevant)

Not full:$\mathbb{Z}/2\mathbb{Z}\mathrm{CAT} \subset \mathbb{Z}/4\mathbb{Z}\mathrm{CAT}, \mathrm{SYM} \subset 1\mathrm{COB}$, full:$\mathrm{GROUP} \subset \mathrm{MONOID}$

Reflective

$Y \xrightarrow{r} X$ such that



- $X, X' \in C$, $Y \in D$
- The above is called a C-reflection of Y
- C is a reflective subcategory of D if all Y have A-reflections
- Examples (quotients and completions)
  - cGROUPS $\subset$ GROUPS; reflection: $G \to G/[G, G]$
  - cMET $\subset$ MET; reflection: completion
- We will return to these as soon as we have seen adjoint functors

Thank you for your attention!

I hope that was of some help.